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Using a generalized model for air traffic delay: An application of information based duration analysis

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ABSTRACT

Airlines air traffic delays cause discomfort to passengers and cost airlines dearly, thus there is no wonder that a growing number of authors from different disciplines have studied air traffic delays and their patterns. This paper departs from existing literature by assuming air traffic delay to be a duration variable whose true distribution is unknown. It suggests a general model that includes several other models as subfamilies and utilizes an information contents based approach to find the most appropriate model to study air traffic delays. The results for two different airlines, American Airlines and United Airlines, reveal that airlines of comparable size and market influence air traffic delays could follow different patterns.

1. Introduction

This essay attempts to address the problem of searching for a suitable mathematical distribution function for flight delays. It suggests using an information content based search criteria to choose the best fit for air traffic delay based on the information contents of data. More specifically, I consider the generalized gamma (*GG*) distribution for modeling air traffic delays. The *GG* family includes many duration distributions such as exponential, gamma and Weibull as subfamilies. This essay illustrates the applications of information functions developed for *GG* family in Dadpay et al. (2007), using data on a sample of flight delays data.

Air traffic delays are both a major source of passengers' complaints and a topic of discussion for authors of different disciplines studying the aviation industry. Thus, it is no surprise that recent years have witnessed an increase in the number of experts investigating this issue. Mayer and Sinai (2003), Mazzeo (2003) and Rupp et al. (2003) likewise study the relationship between service quality and competition in airline markets, considering air traffic delays as a signal of service quality. While Mayer and Sinai find that delays are longer in more competitive markets, Mazzeo and Rupp et al. conclude the opposite. Kostiuk, et al. (2000) study impacts of air traffic delays on the costs to air traffic control systems, airlines and airports. They conclude that an increase in the air traffic delays reduces aircraft productivity, which can prevent airlines from reaching their financial goals. Suzuki (2000) shows that market shares are positively correlated with on-time airline performance, and thus an airline with too many air traffic delays is less likely to retain its market share. Forbes (2008) points out that air traffic delays affect the demand for air travel as well as its costs.

The discrepancy between scheduled departure time and actual departure time and delays in the air and arrival delays have been of interest to many who attempt to develop models to predict air traffic delays and air traffic congestion (Odoni et al., 1994; Shumsky, 1997; Idris et al., 2002; Tu et al., 2005). Glockner (1996) explains that air traffic delays happen when demand for airports or airspace surpasses available capacity. He considers utilizing a tactical-optimization model to decrease the negative impact of air traffic delay. However, he concludes that such a model would be extremely complex because of uncertainty in airport-capacity forecasts and airlines' performance. He recommends using congestion management system to reduce the length of delays. The present study demonstrates how using an information based search criteria will reduce such uncertainties by using the best fit to forecast air traffic delays.

Information theoretic measures are used by researchers in various disciplines. Theil (1967) and Zellner (1971) pioneered applications of information theoretic approaches for econometric analysis. Since these ground-breaking works, development of information theoretic methods in econometrics and their applications in various economic fields have become widespread. Recent years have witnessed an increase in utilizing these methods in empirical studies investigating cases and scenarios in the business world. Aktekin and Soyer (2014) use a generalized gamma to study abandonment behavior and timing in call centers, Yalcinkaya et al. (2017) utilize this approach to investigate the optimal timing for product modification in automobile industry. The present article extends the application of information based models to aviation and airport management.

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The following section reviews the methodology used in this study. The third section presents the empirical results when the methodology is applied to air traffic delays data and is followed by concluding remarks.

2. Methodology

I consider traffic delay data $y_1,\ldots,y_n$ as being independent observations from a random variable Y that has a generalized gamma distribution $GG(\alpha,\beta,\lambda)$ with probability density function:

$$f(y|\alpha,\beta,\lambda)=\frac{\beta}{\lambda^{\alpha\beta}\Gamma(\alpha)}y^{\alpha\beta-1}e^{-\left(\frac{y}{\lambda}\right)^{\beta}},\ y\geq 0,\ \alpha,\beta,\lambda>0 \tag{1}$$

where $\Gamma(.)$ is the Gamma function where α and β are shape parameters and λ is the scale parameter.

The GG family, introduced by Stacy (1962), contains several well-known models as subfamilies (Johnson et al., 1994). The subfamilies of GG include exponential ($\alpha = 1$ and $\beta = 1$), Weibull ($\alpha = 1$) and Gamma ($\beta = 1$). Another subfamily of GG is the generalized normal distribution (2α and $\beta = 2$), which includes Half-Normal.

($\alpha = 1/2$) and Rayleigh ($\alpha = 1$).

I will apply two of the information methods developed in Dadpay et al. (2007) to the air traffic delay data. The following moments of GG distribution will be used for information analysis:

$$\mu(\alpha,\beta,\lambda)=E_{GG}(Y|\alpha,\beta,\lambda)=\frac{\lambda\Gamma\left(\alpha+1/_{\beta}\right)}{\Gamma(\alpha)} \tag{2}$$

$$\mu_{\beta}(\alpha,\beta,\lambda)=E_{GG}(Y^{\beta}|\alpha,\beta,\lambda)=\alpha\lambda^{\beta} \tag{3}$$

$$\upsilon(\alpha,\beta,\lambda)=E_{GG}(\log Y)=\log(\lambda)+\frac{1}{\beta}\psi(\alpha) \tag{4}$$

where $\psi(\alpha)={}^{d\log\Gamma(\alpha)}/_{d\alpha}$ is the digamma function.

Shannon entropy of $GG(\alpha,\beta,\lambda)$ is:

$$\begin{aligned}H(GG)&=-\int f(y|\alpha,\beta,\lambda)\log f(y|\alpha,\beta,\lambda)\,dy\\&=-\int_0^{\infty} f(y|\alpha,\beta,\lambda)\log f(y|\alpha,\beta,\lambda)dy=\log\lambda+\log\Gamma(\alpha)+\alpha\\&\quad-\log\beta+\left(\frac{1}{\beta}-\alpha\right)\psi(\alpha)\end{aligned} \tag{5}$$

The entropy of gamma distribution $G(\alpha,\lambda)$ is given by (5) with $\beta = 1$. The entropy of Weibull distribution $W(\beta,\lambda)$ is given by (5) when $\alpha = 1$. The entropy of exponential distribution $E(\lambda)$ is given by (5) with $\alpha = \beta = 1$. The entropy will be used for examining the distributional fit.

Kullback-Leibler discrimination information function between two models in GG family, $GG(\alpha,\beta,\lambda)$ and $GG_0(\alpha_0,\beta_0,\lambda_0)$, is:

$$\begin{aligned}K(GG{:}\,GG_0)&=\int_0^{\infty} f(y|\alpha,\beta,\lambda)\log\frac{f(y|\alpha,\beta,\lambda)}{f(y|\alpha_0,\beta_0,\lambda_0)}dy\\&=\log\frac{\varphi_\beta}{\varphi_\lambda^{\alpha\varphi_\beta}}-\log\frac{\Gamma(\alpha)}{\Gamma(\alpha_0)}-\alpha+\mu(\alpha,\varphi_\beta,\varphi_\lambda)\\&\quad+(\alpha\varphi_\beta-\alpha_0)\nu(\alpha,\varphi_\beta,\varphi_\lambda)\end{aligned} \tag{6}$$

where $\phi_\beta=\beta/\beta_0$ is the ratio of shape parameters and $\phi_\lambda=(\lambda/\lambda_0)^{\beta_0}$is the ratio of scale parameters, and $\mu(\alpha,\varphi_\beta,\varphi_\lambda)$and $\nu(\alpha,\varphi_\beta,\varphi_\lambda)$are the mean and geometric mean of $GG(\alpha,\varphi_\beta,\varphi_\lambda)$:

$$\mu(\alpha,\varphi_\beta,\varphi_\lambda)=\frac{\varphi_\lambda\Gamma\left(\alpha+1/_{\varphi_\beta}\right)}{\Gamma(\alpha)} \tag{7}$$

$$\nu(\alpha,\varphi_\beta,\varphi_\lambda)=\log(\varphi_\lambda)\frac{1}{\varphi_\beta}\psi(\alpha) \tag{8}$$

It is well known that $K(GG{:}GG_0)\geq 0$; equality holds if and only if $f(y|\alpha,\beta,\lambda)=f(y|\alpha_0,\beta_0,\lambda_0)$ for all $y\geq 0$.

The discrimination information between $GG(\alpha,\beta,\lambda)$ and $G(\alpha_0,\lambda_0)$ is given by (6) with $\phi_\beta=\beta$. The discrimination information between $GG(\alpha,\beta,\lambda)$ and Weibull $W(\beta_0,\lambda_0)$ is given by (6) with $\alpha_0 = 1$. The discrimination information between $GG(\alpha,\beta,\lambda)$ and exponential $E(\lambda_0)$ is given by (6) when $\phi_\beta=\beta$and $\alpha_0 = 1$.

Since $H(GG)$ and $K(GG{:}\,GG_0)$ are functions of the GG parameters, they can be estimated by using the parameter estimates in (5) and (6). I will use two estimation approaches.

2.1. Maximum likelihood estimation (MLE)

The likelihood function based on a set of air traffic delay observations $\mathbf{y}=(y_1,\ldots,y_n)$ from $y\sim f(y|\alpha,\beta,\lambda)=GG(\alpha,\beta,\lambda)$ is:

$$f(y|\alpha,\beta,\lambda)=\left(\frac{\beta}{\lambda^{\alpha\beta}\Gamma(\alpha)}\right)^n\exp\left\{n\left[(\alpha\beta-1)\overline{\log y}-\frac{\overline{y^\beta}}{\lambda^\beta}\right]\right\} \tag{9}$$

where $\overline{y^\beta}=\left(1/_n\right)\sum_{i=1}^n y_i^\beta$and. $\overline{\log y}=\left(1/_n\right)\sum_{i=1}^n \log y_i$

The likelihood equations for the derivatives of log-likelihood function are given by the moment equations (3) and (4) with $\theta_1=\overline{y^\beta}$ and $\theta_2=\overline{\log y}$. The MLE estimates of GG parameters are solutions of these equations $\hat\alpha$, $\hat\beta$and$\hat\lambda$.

Using $\hat\alpha$, $\hat\beta$and $\hat\lambda$ in (5) and (6) gives estimates of the GG entropy $\hat H(GG)$and discrimination information function $\hat K(GG{:}\,GG_0)$. Relationships between $\hat H(GG)$ and the log-likelihood function, the likelihood ratio statistics and information criteria AIC and BIC are shown in Dadpay et al. (2007). The log-likelihood ratio statistic for the two nested models in the GG family is given by

$$LR=2n\left(\hat H_{GG_0}-\hat H_{GG}\right) \tag{10}$$

where $\hat{H_{GG_0}}$ is the ML entropy estimate of the subfamily of GG; gamma when $\alpha_0 = 1$, Weibull $\beta_0 = 1$ and exponential when $\alpha_0=\beta_0 = 1$. The distribution of LR statistic is asymptotically χ_d^2 with d degrees of freedom, where d is the difference between the number of parameters of GG and GG_0 ($d = 1$ for testing gamma and Weibull against GG and $d = 2$ for testing exponential against GG).

The Akaike information criteria AIC and Schwartz information criteria BIC for the GG family are given by:

$$AIC=2n\hat H_{GG}+2k \tag{11}$$

$$BIC=2n\hat H_{GG}+k\log n \tag{12}$$

where k is the number of model parameters ($k = 1$ for exponential, $k = 2$ for Weibull and gamma and $k = 3$ for GG).

Using the MLE estimates of GG and GG_0 parameters in (6) provides information criteria for discriminating between the GG model and its subfamilies.

2.2. Bayesian estimation

Bayesian inference for the GG, $H(GG)$ and $K(GG{:}GG_0)$ are obtained using the prior distribution for the GG parameters used by Dadpay et al. (2007). Given a prior distribution $\pi(\alpha,\beta,\lambda)$, the Bayes Theorem gives posterior distribution:

$$\pi(\alpha,\beta,\lambda|\mathbf{y})\propto\pi(\alpha,\beta,\lambda)f(\mathbf{y}|\alpha,\beta,\lambda) \tag{13}$$

Assuming that α, β and τ parameters are independent, a priori, the following conditional posterior distributions could be obtained:

$$\pi(\alpha|\beta,\lambda,\mathbf{y})\propto[\lambda^{\alpha\beta}\Gamma(\alpha)]^{-n}\exp\{n(\alpha\beta-1)\overline{\log y}\}\pi(\alpha) \tag{14}$$

$$\pi(\beta|\alpha,\lambda,\mathbf{y})\propto\beta^n\exp\left\{n(\alpha\beta-1)\overline{\log y}-\frac{\overline{y^\beta}}{\lambda^\beta}\right\}\pi(\beta) \tag{15}$$