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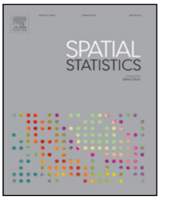

On the estimation of parameters of a spheroid distribution from planar sections

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ABSTRACT

We study two different methods for inferring the parameters of a spheroid distribution from planar sections of a stationary spatial system of spheroids: one method first unfolds non-parametrically the joint size–shape–direction distribution of the observable ellipses in the plane into the joint size–shape–direction distribution of the spheroids followed by a maximum likelihood estimation of the parameters; the second method directly estimates these parameters based on statistics of the observable ellipses using a quasi-likelihood approach. As an application we consider a metal-matrix composite with ceramic particles as reinforcing inclusions, model the inclusions as prolate spheroids and estimate the parameters of their distribution from planar sections.

1. Introduction

Although nowadays various imaging techniques like X-ray computed tomography are available by which three-dimensional specimens, e.g. composite materials, can be represented truly three-dimensional for further analysis this is sometimes too expensive or not applicable due to the type of material, or, the resolution of the sampling technique is too low in order to distinguish very small substructures or inclusions. However, the geometry of such spatial structures still allows for investigation via planar sections where then often more highly resolving sampling techniques are

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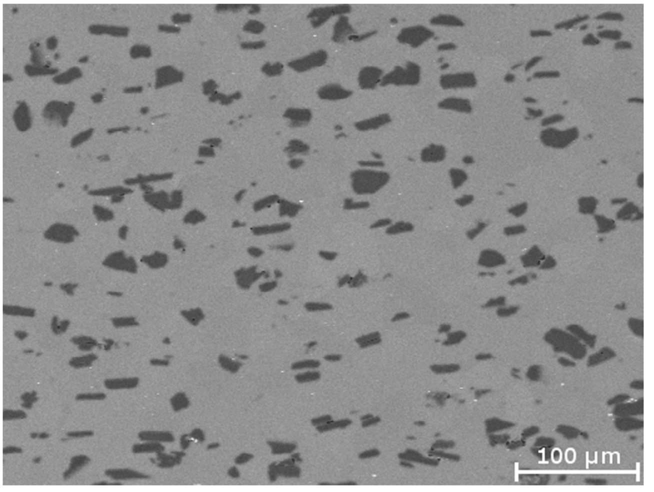

Fig. 1. Planar section (cutout) of a metal-matrix composite with reinforcing ceramic particles resulting from scanning electron microscopy.

available (Nagel, 2010). The methodology aimed to draw inference from information contained in planar sections about the spatial geometrical structure is part of stereology (Ohser and Mücklich, 2000; Nagel, 2010; Chiu et al., 2013).

We were faced with this kind of situation when investigating and modelling metal-matrix composites with ceramic particles as reinforcing inclusions (see Fig. 1) with respect to their fatigue behaviour. The kind of model intended for the fatigue behaviour (Baaske et al., 2018) is based on a model for the spatial configuration of the ceramic inclusions. Besides a model for the spatial arrangement of the inclusions (based on the force-biased algorithm for ellipsoids, see Bezrukov and Stoyan, 2006) this requires the availability of a reasonable model for the distribution of the inclusions in the sense of a grain distribution (Chiu et al., 2013, Sect. 6.5.1). Under the assumption of spatial stationarity it is justified to speak of the *typical* inclusion, i.e., the distribution of which does not depend on the particular (stationary) spatial arrangement of the inclusions (cf., e.g., Schneider and Weil, 2008, Sect. 4.2); likewise this applies for the distribution of the typical section profile of those inclusions which are intersected by some plane. With this reasoning in mind and since in the present paper we solely focus on the distribution of the typical inclusion we work throughout the paper with a stationary *Poisson* particle process (Schneider and Weil, 2008, Sect. 4.1) for the spatial arrangement. We do this even when processing the data (where the Poisson assumption is certainly not true), which is an established approach in stereology (Chiu et al., 2013, p. 437).

As a trade-off between a minimum of information on size, shape and direction and a more realistic assumption on the grain distribution involving typically a larger number of parameters we decided to model the typical inclusion as a random *ellipsoid* with randomness in size, shape and orientation. The corresponding planar section profiles are then *ellipses*. For given data their sizes, shapes and directions (in the plane) can be estimated from digital images of the observable planar sections.

Dating back to Wicksell (1926) the respective stereological objective is to infer the joint distribution of the ellipsoids' size and shape (and also orientation) from the ellipses observable in the planar section. This turned out to be solvable completely and uniquely only in case the ellipsoids are all either prolate or oblate spheroids (ellipsoids of revolution), see the pioneering work of Cruz-Orive (1976). For the case that, besides sizes and shapes, also directions are aimed to be included, the general solution of unfolding the joint size–shape–direction distribution of the observable ellipses in the plane into the joint size–shape–direction distribution of either prolate or oblate spheroids was given by Beneš et al. (1997) and Beneš and Krejčíř (1997), see also Beneš and Rataj (2004, Ch. 6). Hence, from the empirical 3D joint size–shape–direction distribution, typically available as a histogram, one