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Detectability of linear systems over a principal ideal domain with unknown inputs

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ABSTRACT

Observability and detectability conditions are obtained for a sort of linear time invariant systems affected by unknown inputs. It is assumed that the system can be described using matrices whose elements are within a principal ideal domain (PID). It is shown that, under suitable hypothesis, the obtained conditions are necessary and sufficient. The analysis is carried out by making use of the Smith normal form of matrices over a PID. The obtained conditions are a generalization of known conditions for particular sorts of systems which are included in those considered here.

1. Introduction

Antecedents. The problem of estimating the state variables of a system has been one of the main problems studied in control theory. The observability problem essentially consists in finding the conditions under which the trajectories of the state vector of the system can be reconstructed by means of the knowledge of the system output. Such a problem has been successfully solved for several sort of dynamic systems, including linear systems, nonlinear systems, systems with delays, systems with partial differential equations (see, e.g. [1–5]). A more general problem is that of detectability which considers also the case when the reconstruction of the state trajectories cannot be carried out in finite time, but can be done asymptotically. For that case there exists famous Kalman's decomposition, for linear time invariant systems, in which we can see the part of the state that is observable and the part that is not. A generalization of the problem has been carried out by considering that the system under study is affected by unknown inputs. In such a case, for linear systems, observability and detectability conditions were found in [6,7]. In [8], for linear implicit systems, necessary and sufficient observability and detectability conditions were obtained. For the nonlinear case, in [9] conditions were given under which there exists an observer allowing for the state estimation. Hence, based on the recent works one may realize that the state estimation of systems with unknown inputs is still an active topic of research.

On the other hand, representing dynamic systems over rings has been found to be useful since several decades ago [10–12]. As it is stated by Sontag in [10], there is a variety of real systems that may be represented using rings. Furthermore, as explained in the just mentioned paper, the use of rings has served to understand better the linear systems. A brief history and survey for linear systems over rings can be found in [12]. Recently the use of rings has been extended to nonlinear systems also, see, e.g., [13]. In [14] some algebraic structures of nonlinear systems over rings obtained by immersion are studied. In [15] the realization of discrete-time nonlinear input–output equations is addressed by using a non-commutative polynomial ring. The global observability of polynomial nonlinear systems is tackled in [16] using the localization of a polynomial ring. A particular class of dynamic system that can be represented over a ring is that of systems with commensurate delays, there the delays can be grouped together, which allows for considering the coefficients of the mathematical model within a polynomial ring, in case of linear systems (see, [17]), and within a non-commutative ring, in case of nonlinear systems [18]. There are some other systems that can be represented as linear systems over a principal ideal domain. For example, 2-D systems [19,20], linear systems with commensurate delays [21], systems over the integers [22], parametrized system [23]. It is also worth to mention that some computational algebra tools to work with linear systems over rings have been given in [24].

Contribution and methodology. The main contribution of this paper is the obtention of testable conditions under which a linear system over a principal ideal domain (PID) affected by unknown inputs can be said to be observable or detectable, respectively.

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Furthermore, the obtained results are a generalization of already known conditions for linear systems (over a real field) and for linear systems with commensurate delays, both with unknown inputs.

The method used along the paper is built upon the algorithm used by Molinari in [6]. Thus, a chain of matrices are generated recursively. It is shown that such a chain of matrices over a PID is finite. Then the Smith form of the ultimate matrix obtained is examined to get the information that allows to check if the system is observable. As for the detectability, a coordinate transformation is carried out partitioning the system in a sort of Kalman's form, a part being observable and the other part unobservable. Based on this decomposition testable conditions are obtained.

Notation. The set of real numbers (real field) is denoted by $\mathbb{R}$. The ring of integers is denoted by $\mathbb{Z}$. The set of non-negative integers is denoted by $\mathbb{N}$. Given a ring $\mathfrak{R}$, $\mathfrak{R}^{n\times m}$ denotes the set of matrices of n by m dimension whose elements belong to $\mathfrak{R}$. By $\mathfrak{R}^n$ is denoted the free $\mathfrak{R}$-module formed by column vectors of n dimension. $\mathfrak{R}^{1\times n}$ is the free $\mathfrak{R}$-module formed by row vectors of n dimension. An invertible element of $\mathfrak{R}$ is called a unit. By $\operatorname{diag}(a_1, a_2, \ldots, a_r)$ is denoted a square matrix of r by r dimension whose elements are $a_{ii} = a_i\,(i = 1, \ldots, r)$ and $a_{ij} = 0$ for $j \neq i\,(j = 1, \ldots, r)$.

Structure of the paper. The sort of systems studied in this work are described in Section 2, there the observability and detectability definitions are given. In Section 3 preliminary results are given, a Molinari-like algorithm is presented for the case under study. Main results, dealing with the obtained observability and detectability conditions, are exposed in Section 4. Two examples of two systems of a different class are given in Section 5.

2. System description and observability concepts

The systems (over a PID $\mathfrak{R}$) to be considered are those whose dynamics is governed by the following equations

$$\eth x(t) = Ax(t) + Bu(t) \tag{1a}$$

$$y(t) = Cx(t) + Du(t) \tag{1b}$$

where the time t belongs to $\mathbb{S}$, a closed set of the set $\mathbb{T} \in \{\mathbb{R}, \mathbb{Z}\}$. For the continuous time case (when $\mathbb{T} = \mathbb{R}$), $\eth$ should be understood as $\eth x(t) = \dot{x}(t)$, and, for the discrete time case (when $\mathbb{T} = \mathbb{Z}$), $\eth x(t) = x(t+1)$. Thus, for $\mathbb{T} = \mathbb{R}$, $x(\cdot)$ belongs to the $\mathfrak{R}$-module $\mathscr{X}$ of differentiable functions mapping from $\mathbb{S} \subset \mathbb{R}$ to $\mathbb{R}^n$, $u(\cdot)$ belongs to the $\mathfrak{R}$-module $\mathscr{U}$ of continuous functions mapping from $\mathbb{S} \subset \mathbb{R}$ to $\mathbb{R}^m$, and $y(\cdot)$ belongs to the $\mathfrak{R}$-module $\mathscr{Y}$ of continuous functions mapping from $\mathbb{S} \subset \mathbb{R}$ to $\mathbb{R}^p$. For $\mathbb{T} = \mathbb{Z}$, $x(\cdot)$, $u(\cdot)$, and $y(\cdot)$ are elements of the $\mathfrak{R}$-modules $\mathscr{X}$, $\mathscr{U}$, and $\mathscr{Y}$ of functions mapping from $\mathbb{S} \subset \mathbb{Z}$ to $\mathbb{R}^n$, from $\mathbb{S} \subset \mathbb{Z}$ to $\mathbb{R}^m$, and from $\mathbb{S} \subset \mathbb{Z}$ to $\mathbb{R}^p$, respectively. Hence, every element of $\mathfrak{R}$ acts as a linear operator on the respective space. In any case, $u(\cdot) \in \mathscr{U}$ is assumed to be **unknown**. The elements of the matrices A, B, C, and D belong to $\mathfrak{R}$.

Throughout the paper, it will be assumed that

A1. For any matrix $G \in \mathfrak{R}^{q\times n}$, $\eth G x = G \eth x$, for every $x \in \mathscr{X}$.

The above assumption is satisfied by linear systems over the real field, with commensurate delays,[1] over integers, parametrized, etc. The observability and detectability definitions that are to be studied further are the following,

[1] A class of systems that may be represented as in (1) is that of linear systems with commensurate delays. The motion of those systems is governed by the equation

$$\dot{x}(t) = \sum_{i=0}^{k} A_i x(t - i\tau) + \sum_{i=0}^{k} B_i u(t - i\tau).$$

By using the shift operator $\nabla : x(t) \mapsto x(t-\tau)$, and defining the matrices (over the polynomial ring $\mathfrak{R} = \mathbb{R}[\nabla]$) $A(\nabla) = \sum_{i=0}^{k} A_i \nabla^i$ and $B(\nabla) = \sum_{i=0}^{k} B_i \nabla^i$, the system with delays can be represented using (1) (see, e.g., Section 2.1.3 in [25]).

Definition 1. System (1) is **unknown input observable** (UIO) if, and only if, $y(t) = 0$ for all $t \in \mathbb{S}$ implies $x(t) = 0$ for all $t \in \mathbb{S}$.

Definition 2. System (1) is **unknown input detectable** (UID) if, and only if, $y(t) = 0$ for all $t \in \mathbb{S}$ implies $x(t) \to 0$ as $t \to \infty$.

In other words, if the system is UIO, then the exact reconstruction of $x(t)$ may be carried out theoretically by using the values of $y(t)$. Likewise for the UID, but in this case the reconstruction may be done only asymptotically. This is the justification to search for conditions under which the system may be UIO and conditions under which the system may be UID.

3. Preliminary results

Let P be a matrix of rank equal to r, with elements in $\mathfrak{R}$. Since $\mathfrak{R}$ is a PID, there exists an invertible matrix T such that P is put into its Hermite form. Thus, one has that

$$TP = \begin{pmatrix} P_1 \\ 0 \end{pmatrix}$$

where P_1 is of full row rank (i.e. $\operatorname{rank} P_1 = \operatorname{rank} P$). Moreover, there exist two invertible matrices U and W that reduce P to its Smith form, i.e.,

$$UPW = \begin{pmatrix} \operatorname{diag}(\psi_1 \cdots \psi_r) & 0 \\ 0 & 0 \end{pmatrix}$$

where the $\{\psi_i\}$ are **nonzero** elements of $\mathfrak{R}$ satisfying

$$\psi_i | \psi_{i+1} \text{ and } d_i = d_{i-1}\psi_i (d_0 = 1)$$

where d_i is the greatest common divisor (gcd) of all $i \times i$ minors of P. The $\{\psi_i\}' s$ are called the invariant factors of P (which are unique up to units), and the $\{d_i\}' s$ are the determinant divisors of P.

Let $\{\Delta_k\}$ be the matrices generated by the following algorithm,

Step 0. Let us define $N_0 = \Delta_0 \triangleq 0$ (dimension $1 \times n$), $G_0 \triangleq C$, $F_0 \triangleq D$.

Step $k+1$. We define T_k as an invertible matrix over $\mathfrak{R}$ which transforms $\begin{pmatrix} \Delta_k B \\ F_k \end{pmatrix}$ into its Hermite form, i.e.,

$$T_k \begin{pmatrix} \Delta_k B \\ F_k \end{pmatrix} = \begin{pmatrix} F_{k+1} \\ 0 \end{pmatrix}.$$

Hence F_{k+1} is by its construction of full row rank. Thus Δ_{k+1} is defined by the identity

$$T_k \begin{pmatrix} \Delta_k B & \Delta_k A \\ F_k & G_k \end{pmatrix} = \begin{pmatrix} F_{k+1} & G_{k+1} \\ 0 & \Delta_{k+1} \end{pmatrix}. \tag{2}$$

Then, the matrix M_{k+1} is generated as follows,

$$N_{k+1} \triangleq \begin{pmatrix} N_k \\ \Delta_{k+1} \end{pmatrix}, \quad \text{for } k \geq 0$$

$$\begin{pmatrix} M_{k+1} \\ \hline 0 \end{pmatrix} \triangleq \left(\begin{array}{c|c} \operatorname{diag}\left(\psi_1^{(k+1)}, \ldots, \psi_{r_{k+1}}^{(k+1)}\right) & 0 \\ \hline 0 & 0 \end{array} \right) = U_{k+1} N_{k+1} W_{k+1} \tag{3}$$

with U_{k+1} and W_{k+1} being invertible matrices over $\mathfrak{R}$ that transform N_{k+1} to its Smith form. Thus, r_{k+1} is the number of invariant factors of N_{k+1}. By (3), M_{k+1} is of dimension equal to r_{k+1} by n, and has rank equal to r_{k+1}, for every $k \geq 1$. As we will prove it below, the matrix M_k is independent of the choice of the matrices T_i, U_i, and W_i for $i = 1, \ldots, k$.

The above manner of generating Δ_k and M_k is built upon the algorithms given in [26] and latter in a different version in [6].