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Sampled-data consensus in multi-agent systems with asynchronous hybrid event-time driven interactions[☆]

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ABSTRACT

This paper investigates sampled-data consensus in an undirected network of multiple integrators and characterizes the effectiveness of a hybrid event-time driven consensus protocol in different asynchronous scheduling schemes of event detection in terms of interaction topology, asynchronous matrix, and time delays. The proposed hybrid driven protocol has the benefit of guaranteed performance at reduced communication and computation costs and has robustness against interaction/event-detection time delays. Furthermore, the obtained results are still valid in many other practical situations, such as sampled-data consensus with measurement errors and quantized consensus.

1. Introduction

Asynchronous individual dynamics is an important feature in large scale networks of multiple agents, and thus designing decentralized asynchronous coordinating protocols is of particular interest in both theoretical studies and engineering applications. Asynchronous sampled-data consensus, as such a research topic, has been an active research area for several years.

Asynchronous sampled-data consensus was originally addressed by Lin et al. in the setup of multi-agent rendezvous [1]. Rendezvous control aims to drive all agents to meet at a specific point; in other words, it is to make all agents reach a consensus on positions. In [1], each agent was assumed to be able to continuously track the positions of all other agents within its sensing region and compute its way-points over a sequence of time intervals, uncorrelated with others. With several additional assumptions on registration intervals and sensing periods, the studied system attained a property similar to the symmetry of neighboring relationship in synchronous cases. The procedure of "analytic synchronization" was presented for convergence analysis. Also by the concept of "analytic synchronization", Cao et al. investigated an asynchronous sampled-data version of the Vicsek model, where each agent sampled the headings of its neighbors at some discrete event times and changed its heading from one way-point to the other in a monotonic and piecewise-continuous manner [2]. Based on nonlinear paracontractions theory, Fang and Antsaklis studied an asynchronous discrete-time consensus model and established a convergence result on state consensus under directional and time-varying topologies [3]. From the above results, it can be seen that all agents perform either periodic data sampling or aperiodic but time-driven data sampling with bounded periods. The same time-driven style of data sampling was also the basis of many results on double-integrator networks, see [4–6]; but it raises a problem on how to remove unnecessary data sampling at scheduled event times.

The event-driven control is another technique widely used in scheduling data-sampling actions. It has favorable advantages over the pure time-driven control in applications with regard to communication costs [7–13]; but it is more theoretically challenging in convergence analysis and also has difficulty in ensuring a lower bound of inter-event times in protocol design [14]. For the single-integrator consensus problem in undirected networks, Dimarogonas et al. designed several event-driven controllers, whose updates depended on the ratio of a certain measurement error with respect to the norm of a state function; to avoid continuous monitoring of measurement errors, these control laws were further revised by a self-triggering approach [15]. Also with the aim of relaxing the

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requirement of continuous monitoring of neighbors' states, Seyboth et al. proposed a consensus strategy based on exponentially decreasing event-triggering thresholds in time with nonnegative offset for networks of single-integrators with and without communication delays, and for networks of double-integrators [16]. In their strategy, each agent broadcasted its state whenever the difference between its current state and its latest broadcasted state exceeded its triggering threshold, and updated its controller whenever it sent or received a new measurement. The authors also analyzed the lower bound of inter-event times, which was related to the initial states of agents. In [17], the authors used the measurement error of a convex combination of neighbors' states rather than the measurement error of agents' own states to design an event-based protocol for rendezvous and showed that lower bounds of inter-event times of each agent were state-dependent. Based on algebraic Riccati equations, event-based consensus control for general linear agents has also been studied recently [18,19]. In [18], the authors managed to define an event-triggering function, which avoided continuous communication between neighboring agents; and in [19], the authors tried the addition of a positive constant in event-triggering thresholds to guarantee positive inter-event times in some particular cases.

This paper aims to solve the asynchronous consensus problem in the framework of edge-event based sampled-data consensus. Edge-event based consensus was previously addressed in fixed undirected networks in [20], and then revisited in both bidirectional networks and leader-following networks in the scenarios of continuous event detection and synchronous periodic event detection in [21]. In [22], the events on edges were considered in the synchronization of nonlinear dynamical agents with guaranteed lower bounded inter-event times. In the edge-event based framework, each information link is assigned a sequence of edge events, which activate the mutual data sampling and controller updates of the two linked agents. The idea of independent treatment of information links was previously indicated in [23]. It has the advantage of reduced communication costs and serves as an alternative to the traditional scheme in which data-sampling events are defined with respect to agents, and each event triggers the communication of its associated agent with all its neighbors [15–19]. In [21], event-detecting rules were developed to ensure a lower bound of inter-event times in the case of continuous event detection. In [24,25], a class of edge-event based consensus protocols was given for undirected networks with synchronous periodic event detection and time-varying delays; and sufficient conditions in terms of maximum allowable time delay were given for consensus control. In this paper, we also study a time-driven fashion of event-detection scheduling; but we focus on the case of asynchronous periodic and aperiodic event detection, and characterize the effectiveness and delay robustness of a class of hybrid event-time driven consensus protocols. The asynchronous feature of multi-agent systems and aperiodic event detection differentiate this paper from [12] in basic model setups. In the latter paper, a strategy of periodic event-triggered control was presented for linear systems. The choice of time-driven scheduling of event detection relaxes the requirement of continuous state monitoring, and reduces the burden of event detectors; furthermore, it can naturally ensure a state-independent lower bound of inter-event times and eliminate Zeno behavior of data sampling.

The contributions of this paper are threefold. First, it presents several event-detecting rules and gives relaxed conditions in terms of interaction topology, asynchronous matrix, and time delays for consensus solvability in different asynchronous scheduling schemes of event detection. This paper also shows the consensus robustness against interaction time delays and the advantage in reducing communication and computation costs. Second, this paper provides a general theoretical approach to decide conditions, under which we could revise continuously event-monitoring protocols with guaranteed effectiveness by adding a period of rest time after each data-sampling event. This revision brings a lower bound of inter-event times and makes considered protocols more applicable in engineering. Third, the obtained results and analysis techniques are valid in the traditional time-driven sampled-data consensus with asynchronous interactions and also applicable in other settings, such as in the presence of measurement errors or quantized communications [26,27].

This paper is organized as follows. The problem is formulated in Section 2. The asynchronous periodic and aperiodic event detection is studied in Sections 3 and 4, respectively. Application examples and simulations are given in Sections 5 and 6, respectively. Finally, the paper is concluded in Section 7. In the Appendix, some preliminary concepts and lemmas are given for reference.

Notation. $\lfloor\theta\rfloor$ gives the largest integer not greater than θ; $\mathbf{1}$ denotes the column vector $[1\ 1\ \ldots\ 1]^T$ with a compatible dimension; $\circ$ stands for the entrywise product (Hadamard product) of matrices; $T = \operatorname{diag}([\xi_1\ \xi_2\ \ldots\ \xi_m])$ denotes the diagonal matrix with ξ_i in the (i,i)-th diagonal position; if T is nonnegative, $T^{1/2} = \operatorname{diag}([\xi_1{}^{1/2}\ \xi_2{}^{1/2}\ \ldots\ \xi_m{}^{1/2}])$; and with an abuse of notation, $T^{-1/2} = \operatorname{diag}([\zeta_1\ \zeta_2\ \ldots\ \zeta_m])$, defined by

$$\zeta_i = \begin{cases} \xi_i^{-\frac{1}{2}}, & \text{if } \xi_i > 0 \\ 0, & \text{otherwise.} \end{cases}$$

2. Problem formulation

The studied multi-agent system is composed of n single-integrators. They are labeled with 1 through n and take the following dynamics:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \ldots, n,$$

where $x_i(t) \in \mathbb{R}$ denotes the state of agent i, and $u_i(t)$ is a state feedback, called *protocol*, to be designed based on the local information received by agent i from its neighbors. The information links among agents are assumed to be bi-directional and modeled by the edges of an undirected simple graph $\mathcal{G}$ with n vertices $v_1, v_2, \ldots, v_n$. In $\mathcal{G}$, vertex v_i represents agent i, $i = 1, 2, \ldots, n$; the existence of an edge (v_i, v_j) implies an effective information link connecting agent i with agent j. Let m be the total number of edges in $\mathcal{G}$ and for notational simplicity, denote them by $e_1, e_2, \ldots, e_m$. For each edge e_p, there exists a pair of adjacent agents i and j, such that $e_p = (v_i, v_j)$. In such a case, we say that v_i and v_j are *incident* to edge e_p, denoted by $i, j \sim e_p$.

For each link e_p with $i, j \sim e_p$, agents i and j collectively generate a sequence of time instants $t_0^p, t_1^p, t_2^p, \ldots$, at which they check the triggering conditions of edge events of e^p. If the conditions are satisfied, agents i and j sample the relative state between them and update their controllers. Let function $k^p(t)$ index the most recent edge-event time in $t_0^p, t_1^p, t_2^p, \ldots$ at time t; mathematically,

$$k^p(t) = \max\Big\{k : t_k^p \le t, i, j \sim e_p, \text{ an edge-event between agents } i \text{ and } j \text{ occurs at } t_k^p\Big\}.$$

Now, we propose the following protocol[1]:

$$u_i(t) = \sum_{p,j:i,j\sim e_p} w_p\Big(x_j(t^p_{k^p(t)}) - x_i(t^p_{k^p(t)})\Big), \quad i = 1, 2, \ldots, n, \tag{1}$$

[1] $\sum_{p,j:i,j\sim e_p} = \sum_{j\in\{j:\exists p, \text{ s.t. } i,j\sim e_p\}} \sum_{p\in\{p:i,j\sim e_p\}}$, where $\{p : i, j \sim e_p\}$ is in fact a singleton when i and j are given.