Systems & Control Letters 89 (2016) 61–65

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Leader-following consensus for a class of second-order nonlinear multi-agent systems[☆]

Chuanrui Wang [a,*], Xinghu Wang [b], Haibo Ji [a]

[a] Department of Automation, University of Science and Technology of China, Hefei, Anhui 230027, PR China

[b] Key Laboratory of Systems and Control, Chinese Academy of Sciences, Beijing 100190, PR China

ARTICLE INFO

Article history:
Received 29 March 2014
Received in revised form
11 July 2015
Accepted 9 December 2015
Available online 18 January 2016

Keywords:
Multi-agent systems
Distributed control
Nonlinear systems
Directed topology

ABSTRACT

This paper deals with the leader-following consensus problem for a class of multi-agent systems with nonlinear dynamics and directed communication topology. The control input of the leader agent is assumed to be unknown to all follower agents. A distributed adaptive nonlinear control law is constructed using the relative state information between neighboring agents, which achieves leader-following consensus for any directed communication graph that contains a spanning tree with the root node being the leader agent. Compared with previous results, the nonlinear functions are not required to satisfy the globally Lipschitz or Lipschitz-like condition and the adaptive consensus protocol is in a distributed fashion. A numerical example is given to verify our proposed protocol.

1. Introduction

Cooperative control of multi-agent systems has gained much attention due to its broad potential applications in many engineering systems, such as unmanned air vehicle formations, robotic teams, sensor networks, or satellite clusters. One topic in multi-agent systems is the consensus problem that aims to design distributed control laws such that the states or outputs of all agents reach an agreement.

Recently, many researchers have studied the consensus problem of linear multi-agent systems. [1] studied the average consensus problem of first-order multi-agent systems with switching directed communication graphs or time-delays. [2,3] studied the second-order leader-following consensus problem. [4] proposed a unified way to deal with the consensus problem of general linear multi-agent systems. [5] studied the leader-following consensus problem of linear multi-agent systems, where both state and output feedback control laws were designed based on linear quadratic regulator (LQR) optimal control approach. [6] proposed distributed control laws for general linear multi-agent systems with a leader whose control input is nonzero and not available to any follower. [7] and [8] investigated the output consensus problem of heterogeneous linear multi-agent systems.

In reality, all physical systems are inherently nonlinear, such as robot system, aircraft system and induction motor system [9]. In the existing literature, there are only few results about the consensus problem of nonlinear multi-agent systems. [10–16] investigated the consensus problem of a class of nonlinear multi-agent systems in the strict feedback form, where the nonlinear functions were assumed to satisfy the globally Lipschitz or Lipschitz-like condition. [17] proposed a continuous robust consensus tracking protocol for an integrator-type multi-agent system with disturbances and unmodeled dynamics, but the control gains depend on the initial conditions. By utilizing the agent's own state information and the relative state information between neighboring agents, [18] investigated the consensus problem of second-order nonlinear multi-agent systems with parametric uncertainties, while [19,20] studied the consensus problem for a class of nonlinear multi-agent systems in strict feedback form. [21] proposed robust neural adaptive control algorithms for the cooperative tracking control of higher-order integrator heterogeneous multi-agent systems with unknown nonlinear dynamics and unknown disturbances. However, in some cases, as in most of the existing literature, only the relative measurement information is available to design control laws, and the agent's own measurement information is not available. On the other hand, the control protocols developed in [10–14,16–18] further required that each agent has to know some properties of the entire communication graph, such as the general algebraic connectivity of the information-exchange matrix. Note that this is global information which, in general, is not available to each agent. It should be pointed out that [15] have proposed

☆ This work was supported by the National Natural Science Foundation of China under Grant 61273090.

* Corresponding author.
E-mail address: hugh@mail.ustc.edu.en (C. Wang).

http://dx.doi.org/10.1016/j.sysconle.2015.12.007
0167-6911/© 2016 Published by Elsevier B.V.

adaptive consensus protocols without using any global information, but they were only applicable to undirected communication topology. Under directed communication topology, we proposed a distributed adaptive leader-following consensus protocol for a class of integrator-type nonlinear multi-agent systems in [22].

In this paper, we further study the leader-following consensus problem for a class of second-order nonlinear multi-agent systems under directed communication topology. The control input of the leader agent is assumed to be active and unknown to all follower agents. By choosing an appropriate Lyapunov function, a distributed adaptive control law is constructed that solves the leader-following consensus problem for any directed communication graph that contains a spanning tree with the root node being the leader agent. The main contributions of this paper are composed of three aspects. First, compared with [10–16], we consider a class of more general nonlinear multi-agent systems, where neither the globally Lipschitz nor Lipschitz-like condition is imposed on the nonlinear functions. Therefore, an intrinsic nonlinear rather than linear control law is proposed in this paper. Second, different from [10–14,16–21], our consensus protocol is in a distributed fashion without using any global information, such as the eigenvalues of the corresponding Laplacian matrix or the agent's own state information. Moreover, the communication topology we considered is directed rather than undirected as considered in [15]. Third, contrary to [12,14–16], where the control input of the leader agent is assumed to be either equal to zero or available to all follower agents, we assume that the control input of the leader agent can be nonzero and unknown to all follower agents.

2. Preliminaries

We first introduce some basic concepts and notations in graph theory that can be found in [23].

Let $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a directed graph of order N, where $\mathcal{V} = \{1, , 2, \ldots, N\}$ is the set of nodes, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of directed edges and $\mathcal{A} = [a_{ij}]_{N\times N}$ is the weighted adjacency matrix with $a_{ii} = 0$. A directed edge denoted by the pair (j, i) represents a communication channel from j to i. The neighborhood of the ith agent is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. For any $i, j \in \mathcal{V}$, $a_{ij} \geq 0$ and $a_{ij} > 0$ if and only if $j \in \mathcal{N}_i$. The Laplacian matrix of the directed graph $\mathcal{G}$ is defined as $L = D - \mathcal{A}$, where $D = \text{diag}(d_1, d_2, \ldots, d_N)$ is called the degree matrix of $\mathcal{G}$ with $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, $i = 1, \ldots, N$. A directed path from node i_1 to node i_k is a sequence of ordered edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$, where $i_j \in \mathcal{V}$, $(i_l, i_{l+1}) \in \mathcal{E}$, $j = 1, \ldots, k$, $l = 1, \ldots, k-1$. A directed graph has a spanning tree, if there is a node (called the root), such that there is a directed path from the root to every other node in the graph. A subgraph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1, \mathcal{A}_1)$ of $\mathcal{G}$ is an induced subgraph if $\mathcal{V}_1 \subseteq \mathcal{V}$ and $\mathcal{E}_1 \subseteq \mathcal{E} \cap (\mathcal{V}_1 \times \mathcal{V}_1)$.

3. Problem statement

Consider a class of nonlinear multi-agent systems with $N + 1$ agents as follows

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= u_i + f(x_i), \quad i = 0, \ldots, N \end{aligned} \tag{1}$$

where $x_i = [x_{i1}, x_{i2}]^T \in \mathbb{R}^2$ represents the state and $u_i \in \mathbb{R}$ represents the control input of agent i. $f : \mathbb{R}^2 \to \mathbb{R}$ is a continuously differentiable function. The agent indexed by 0 is called the leader, and the rest agents indexed by $1, \ldots, N$ are referred as the followers.

Consider a directed graph $\bar{\mathcal{G}}$ consisting of N follower agents and a leader agent. The information exchange between different follower agents is represented by a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ of order N, which is a subgraph of directed graph $\bar{\mathcal{G}}$. The elements of the weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{N\times N}$ satisfy that $a_{ii} = 0$, $a_{ij} \geq 0$ and $a_{ij} = 1$ if and only if node i can receive the information of node j, i.e., $(j, i) \in \mathcal{E}$. The leader adjacency matrix is defined as a diagonal matrix $B = \text{diag}(b_1, b_2, \ldots, b_N)$, where $b_i = 1$ if the follower agent i has access to the leader's state x_0 and $b_i = 0$ otherwise. In most cases, the leader's state information x_0 cannot be available to all follower agents but to only a part of the follower agents. The directed graph $\bar{\mathcal{G}}$ is assumed to satisfy the following assumption.

Assumption 1. The directed communication graph $\bar{\mathcal{G}}$ contains a spanning tree with the root node being the leader agent 0.

Denote $H = L + B$. The following lemma presents a useful property of H.

Lemma 1 *([21]). If the directed graph $\bar{\mathcal{G}}$ contains a spanning tree with the root node being the leader agent 0, then H is invertible. Moreover, denote $q = [q_1, \ldots, q_N]^T = H^{-1}\mathbf{1}_N$, and $Q = \text{diag}\left\{\frac{1}{q_1}, \ldots, \frac{1}{q_N}\right\}$, where $\mathbf{1}_N = [1, \ldots, 1]^T$, then Q and $QH + H^TQ$ are positive definite matrices.*

The objective of this paper is to solve the following leader-following consensus problem of system (1).

Definition 1 (*Leader-Following Consensus Problem*)**.** Design distributed control laws $u_i = u_i(z_i)$, $i = 1, \ldots, N$, where z_i denotes the local relative state information between ith agent and its neighboring agents and is defined by

$$z_i = \sum_{j=1}^{N} a_{ij}(x_i - x_j) + b_i(x_i - x_0),$$

such that for any initial conditions $x_i(0)$, the leader-following consensus error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0$$

where $e = [e_1^T, \ldots, e_N^T]^T$ and $e_i = x_i - x_0$.

To facilitate our analysis, the following mild assumption is also needed.

Assumption 2. The leader agent's state x_0 is bounded by an unknown positive constant θ_c, that is $\|x_0\| \leq \theta_c$.

4. Main results

In this section, we will a design distributed control law to solve the leader-following consensus problem of system (1).

Let $z = [z_1^T, \ldots, z_N^T]^T$. Then, we have $z = (H \otimes I_2)e$. By Lemma 1, H is invertible under Assumption 1. Therefore, we only need to design distributed control laws $u_i = u_i(z_i)$, $i = 1, \ldots, N$ such that the relative state z approaches zero asymptotically.

Since neither the globally Lipschitz nor Lipschitz-like condition is imposed on nonlinear function f, the linear protocol $u_i = \alpha z_{i1} + \beta z_{i2}$ proposed in [10,12–14] is not feasible. Instead, the following nonlinear control law is designed

$$\begin{aligned} \dot{\hat{\theta}}_i &= \chi_i(\xi_i)\xi_i^2, \quad \hat{\theta}_i(0) = c_i \\ u_i &= -5\hat{\theta}_i\gamma_i(\xi_i^2)\xi_i - \kappa\,\text{sgn}(\xi_i), \quad i = 1, \ldots, N. \end{aligned} \tag{2}$$

where $\xi_i = z_{i2} + 2z_{i1}$, χ_i is a nonlinear function to be determined, and $\gamma_i \geq 1$ is any nondecreasing function satisfying $\gamma_i^{\frac{1}{4}}(\xi_i^2) \geq \max\left\{\chi_i(\xi_i), |\xi_i|^3\right\}$, $c_i > 0$ is any positive constant. $\kappa > 0$ is an