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Review

On the perturbation of bimodal control linear systems

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ABSTRACT

The aim of this paper is the study of local perturbations of a bimodal system which consists of two linear control systems on each side of a given hyperplane. We follow Arnold's technique based on obtaining a miniversal deformation corresponding to the action of a group associated to a simultaneous feedback equivalence. An application to the study of the controllability of local perturbations of such a system is included.

Contents

1. Introduction

Bimodal systems are a class of switched linear systems which in the last decade has been the object of increasing research mainly in those aspects concerning controllability and stability (see, for instance, [1] and references therein). Some reduced forms have been obtained for this class of systems (see for example [2–4]). However, the stability of this reduced forms under small perturbations has not been considered. For standard time invariant linear systems the perturbation of the corresponding canonical forms has been studied in [5–8] following Arnold's technique. Our aim in this note is to extend the former works to bimodal control linear system (simply bimodal system in what follows). More precisely we focus on the simultaneous feedback equivalence which is the natural extension of the state feedback equivalence of standard linear systems. Our main motivation for considering this equivalence relation is that it preserves controllability. As Arnold showed the versal deformation is an efficient tool for studying local perturbations, as evidenced in the above references. Our contribution in this note is the obtention of a miniversal deformation of a given bimodal system corresponding to the simultaneous feedback equivalence and its application to the study of the controllability of these systems under small perturbations. Specifically we show, by applying the characterization of controllable bimodal systems given in [1], that the set of controllable bimodal systems is open and that, unlikely to what happens in the case where ordinary control linear systems are considered, it is not dense. Similar questions have been

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considered in [9] for planar bimodal systems in the particular case of similarity equivalence.

Let $\mathbb{R}$ be the field of real numbers. We will denote by $\mathbb{R}^{n\times m}$ the set of $n\times m$ matrices with coefficients in $\mathbb{R}$ and by $GL_n(\mathbb{R})$ the group of invertible real matrices of order n. If A and B are two square matrices we write $[A,B]=AB-BA$. In what follows, given $A\in\mathbb{R}^{n\times m}$, A^t denotes the *transpose* of A and if $A\in\mathbb{R}^{n\times n}$, $tr\,A$ denotes the *trace* of A. If $A\in\mathbb{R}^{n\times m}$ is a matrix, we identify A with the linear map $\mathbb{R}^m\longrightarrow\mathbb{R}^n$ defined in a natural way.

Given matrices $A\in\mathbb{R}^{n\times n}$ and $b\in\mathbb{R}^{n\times 1}$ we will denote by $(A\mid b)$ the block matrix formed by matrix A followed by matrix b. We will consider bimodal systems defined by equations of the form

$$\begin{cases}\dot{x}(t)=A_1x(t)+bu(t) & \text{if } c^tx(t)\le 0\\ \dot{x}(t)=A_2x(t)+bu(t) & \text{if } c^tx(t)>0\end{cases} \tag{1}$$

where $b\in\mathbb{R}^{n\times 1}$ and $A_i\in\mathbb{R}^{n\times n}$, $i=1,2$, are such that A_1,A_2 coincide on the hyperplane $\mathcal{V}=\ker c^t$, that is, $A_{1|\mathcal{V}}=A_{2|\mathcal{V}}$. We can assume without loss of generality that $c^t=(0,\dots,0,1)$. We denote the above bimodal system by $(A_1,\ A_2,\ b)$.

We consider, in the set of above systems, simultaneous feedback equivalence defined by $((A_1\mid b),(A_2\mid b))\sim((A_1'\mid b'),(A_2'\mid b'))$ where

$$\begin{aligned}(A_i'\mid b')&=S(A_i\mid b)\begin{pmatrix}S^{-1}&0\\ f&\tau\end{pmatrix}\\ &=(SA_iS^{-1}+Sbf\mid Sb\tau)\quad i=1,2\text{ with }S(\mathcal{V})=\mathcal{V}\end{aligned} \tag{2}$$

for some $S\in GL_n(\mathbb{R}), f\in\mathbb{R}^{1\times n}, \tau\in\mathbb{R}$.

Then, we obtain, following Arnold's technique, a miniversal deformation of a given bimodal system with regard to the above equivalence relation, which allows to study the behavior of controllability under local perturbations of the system.

2. Preliminaries

As said in the Introduction, a bimodal system consists of two subsystems, defined by matrices $(A_1,\ A_2,\ b)$ with $A_{1|\mathcal{V}}=A_{2|\mathcal{V}}$.

Let $\mathcal{M}=\{((A_1\mid b),(A_2\mid b));A_{1|\mathcal{V}}=A_{2|\mathcal{V}}\}$ and

$$\mathcal{G}=\left\{\begin{pmatrix}S&0\\ f&\tau\end{pmatrix};\ S\in GL_n(\mathbb{R}),\ S(\mathcal{V})=\mathcal{V},f\in\mathbb{R}^{1\times n},\tau\in\mathbb{R},\tau\neq 0\right\}. \tag{3}$$

Notice that if $e_i=(0,\dots,0,1,0,\dots,0), i=1,\dots,n$, then $e_1,\dots,e_{n-1}$ is a basis of $\mathcal{V}=\ker(0,\dots,0,1)^t$ and if $(x_1,\dots,x_n)\in\mathcal{V}$, then $x_n=0$. Since $S(\mathcal{V})=\mathcal{V}$, the last component of $S(e_i), i=1,\dots,n-1$ must be 0 and therefore $S=\begin{pmatrix}S_{11}&s^1\\ 0&s\end{pmatrix}$, with $S_{11}\in GL_{n-1}(\mathbb{R}), s^1\in\mathbb{R}^{(n-1)\times 1}$ and $s\neq 0$. Observe also that $\mathcal{G}$ is a subgroup of $GL_{n+1}(\mathbb{R})$ and it can be identified with an open set of $\mathbb{R}^{n^2+2}$. Hence, for any $\mathcal{P}\in\mathcal{G}$, the tangent space $T_{\mathcal{P}}\mathcal{G}$ is the set of matrices $\mathcal{P}=\begin{pmatrix}P&0\\ p_1&q\end{pmatrix}$ with $P=\begin{pmatrix}P_{11}&p^1\\ 0&p\end{pmatrix}$, where $P_{11}\in\mathbb{R}^{(n-1)\times(n-1)}, p^1\in\mathbb{R}^{(n-1)\times 1}, p\in\mathbb{R}$ and $p_1\in\mathbb{R}^{1\times n}, q\in\mathbb{R}$.

We consider in $\mathcal{M}$ the inner product defined by

$$\begin{aligned}&\langle((A_1\mid b),(A_2\mid b)),((A_1'\mid b'),(A_2'\mid b'))\rangle\\ &\quad=tr(A_1(A_1')^t+b(b')^t+A_2(A_2')^t+b(b')^t)\end{aligned} \tag{4}$$

and the action of $\mathcal{G}$ on $\mathcal{M}$ given by

$$\begin{aligned}&\alpha\left(\begin{pmatrix}S&0\\ f&\tau\end{pmatrix},((A_1\mid b),(A_2\mid b))\right)\\ &\quad=\left(S(A_1\mid b)\begin{pmatrix}S^{-1}&0\\ f&\tau\end{pmatrix},\ S(A_2\mid b)\begin{pmatrix}S^{-1}&0\\ f&\tau\end{pmatrix}\right)\end{aligned} \tag{5}$$

$$\quad=((SA_1S^{-1}+Sbf\mid Sb\tau),(SA_2S^{-1}+Sbf\mid Sb\tau)). \tag{6}$$

The following notation is also used:

$$\begin{aligned}&\alpha\left(\begin{pmatrix}S&0\\ f&\tau\end{pmatrix},((A_1\mid b),(A_2\mid b))\right)\\ &\quad=\begin{pmatrix}S&0\\ f&\tau\end{pmatrix}\cdot((A_1\mid b),(A_2\mid b)).\end{aligned} \tag{7}$$

Given a pair $((A_{10}\mid b_0),(A_{20}\mid b_0))\in\mathcal{M}$, we denote by $\phi:\mathcal{G}\to\mathcal{M}$ the map defined by

$$\phi(\mathcal{S})=\mathcal{S}\cdot((A_{10}\mid b_0),(A_{20}\mid b_0)), \tag{8}$$

with $\mathcal{S}=\begin{pmatrix}S&0\\ f&\tau\end{pmatrix}\in\mathcal{G}$.

Let us denote by $\mathcal{O}_0$ the orbit of $\mathcal{A}_0=((A_{10}\mid b_0),(A_{20}\mid b_0))\in\mathcal{M}$ under the $\mathcal{G}$-action, that is to say, $\mathcal{O}_0=\{\mathcal{S}\cdot\mathcal{A}_0;\ \mathcal{S}\in\mathcal{G}\}$. It is known that the orbit $\mathcal{O}_0$ is a submanifold of $\mathcal{M}$ (see for instance, [5]). Let $T_{\mathcal{A}_0}\mathcal{O}_0$ be the tangent space to $\mathcal{O}_0$ at $\mathcal{A}_0$ and $\mathfrak{B}=(T_{\mathcal{A}_0}\mathcal{O}_0)^{\perp}$. Denote by $\mathcal{I}$ the unit element in $\mathcal{G}$. Then, a miniversal deformation of $\mathcal{A}_0$ can be obtained by applying a theorem of Arnold [10]; (see also [7]). Its statement in our particular set-up is as follows.

Theorem 2.1. *With the above notation, the linear variety $\mathcal{A}_0+\mathfrak{B}$ has the following universal property: Let $\psi:\mathfrak{B}\to\mathcal{M}$ be the map defined by $\psi(\chi)=\mathcal{A}_0+\chi$. Then for any differentiable map $\varphi:\mathbb{R}^N\to\mathcal{M}$ such that $\varphi(0)=\mathcal{A}_0$, there exist a neighborhood U of 0 in $\mathbb{R}^N$, a differentiable map $\eta:U\to\mathfrak{B}$ such that $\eta(0)=0$ and a differentiable map $\xi:U\to\mathcal{G}$ with $\xi(0)=\mathcal{I}$ such that $\varphi(\mu)=\alpha(\xi(\mu),\psi(\eta(\mu)))$.*

The linear variety $\mathcal{A}_0+\mathfrak{B}$ has minimal dimension among those satisfying the universal property in the statement above and it is called a *miniversal deformation* of $\mathcal{A}_0$.

3. Construction of a miniversal deformation

As stated in Theorem 2.1, in order to obtain explicitly a miniversal deformation of $\mathcal{A}_0=((A_{10}|b_0),(A_{20}|b_0))$ we have to characterize the elements in $T_{\mathcal{A}_0}\mathcal{O}_0$.

Lemma 3.1. *For any $\mathcal{A}_0\in\mathcal{M}$, the tangent space $T_{\mathcal{A}_0}\mathcal{O}_0$ is the vector space consisting of elements*

$$\begin{aligned}&(([P,\ A_{10}]+b_0p_1\mid b_0q+Pb_0),\\ &\quad([P,\ A_{20}]+b_0p_1\mid b_0q+Pb_0))\end{aligned} \tag{9}$$

where P, p_1 and q are as in Section 2.

Proof. Since $T_{\mathcal{A}_0}\mathcal{O}_0=\operatorname{Im}(d\phi_{\mathcal{I}})$ (see [5]), it suffices to compute $\phi(\mathcal{I}+\varepsilon\mathcal{P})-\phi(\mathcal{I})$ where $\mathcal{P}=\begin{pmatrix}P&0\\ p_1&q\end{pmatrix}\in T_{\mathcal{I}}\mathcal{G}$ and consider its linear part. Recall that if ε is small enough, $(I+\varepsilon P)^{-1}=I-\varepsilon P+\varepsilon^2P^2-\cdots$ where the right side of this equality is a convergent series in εP. Then, taking into account the definition of ϕ, we have

$$\begin{aligned}\phi(\mathcal{I}+\varepsilon\mathcal{P})&=(((I+\varepsilon P)A_{10}(I-\varepsilon P+\cdots)\\ &\quad+(I+\varepsilon P)\varepsilon b_0p_1\mid(I+\varepsilon P)b_0(\varepsilon q+1)),\\ &\quad((I+\varepsilon P)A_{20}(I-\varepsilon P+\cdots)\\ &\quad+(I+\varepsilon P)\varepsilon b_0p_1)\mid((I+\varepsilon P)b_0(\varepsilon q+1))\\ &=((A_{10}\mid b_0)+\varepsilon(PA_{10}-A_{10}P+b_0p_1\mid b_0q+Pb_0),\\ &\quad(A_{20}\mid b_0)+\varepsilon(PA_{20}-A_{20}P+b_0p_1\mid b_0q+Pb_0)))+o(\varepsilon^2)\end{aligned}$$

and lemma follows easily. □

Then $((A_1\mid b),(A_2\mid b))\in(T_{\mathcal{A}_0}\mathcal{O}_0)^{\perp}$ if and only if

$$\begin{aligned}&\langle((([P,\ A_{10}]+b_0p_1\mid b_0q+Pb_0),\\ &\quad([P,\ A_{20}]+b_0p_1\mid b_0q+Pb_0)),\ ((A_1\mid b),(A_2\mid b))\rangle=0\end{aligned}$$