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Distributed self-tuning consensus and synchronization

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HIGHLIGHTS

- New algorithm for distributed self-tuning synchronization of multi-agent systems.
- Error between velocity of an agent and the average of its neighbors is minimized
- Algorithm generates nonnegative and primitive inter-agent coupling matrix.
- The agent velocities converge toward same constant value.
- Velocities converge sufficiently fast so that distances between agents are bounded.

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ABSTRACT

The problem of self-tuning of coupling parameters in multi-agent systems is considered. Agent dynamics are described by a discrete-time double integrator with unknown input gain. Each agent locally tunes the strength of interaction with neighboring agents by using a normalized gradient algorithm (NGA). The tuning algorithm minimizes the square of the error between an individual agent's state (velocity) and the one step delayed average of its own state and the states of its neighbors. Assuming that the network graph is strongly connected, it is proved that the sequence of coupling parameters is convergent and all velocities converge toward the same constant value.

1. Introduction

Synchronization processes represent a form of emergence in a population of networked systems. This intriguing phenomenon of collective behavior is observed in natural and man made systems in biology, chemistry, physics and engineering, as well as in the arts and socials contexts. Winfree [1] assumed that a rhythmical coherent activity of a group can be modeled by a population of self-sustained and interacting oscillatory elements. One of the most popular models was proposed by Kuramoto [2] who considered a collection of limit-cycle oscillators each running at a different natural frequency, and coupled via a sine function of their phase differences. Generally speaking, Kuramoto oscillators synchronize when individual frequencies lock onto some common value. A comprehensive list of references on the subject of synchronization in oscillatory networks can be found in recent surveys [3–5]. Closely related to and often overlapping with synchronization is the so called consensus problem. As stated in [6], a group of interacting dynamical systems (networked agents) achieves consensus when agreement is reached with respect to a certain variable that depends on the state of all of the agents. One of the first formal considerations of consensus describes how a group of individuals might reach agreement on a common probability distribution by pooling their individual opinions [7]. The work by Vicsek et al. [8] can be considered a motivational paper for many results in the area of consensus and presents a simple model of autonomous agents all moving in a plane with the same speed and different headings. Each agent adjusts its heading based on the average of the neighbors' headings including its own. Jadbabaie et al. [9] present a formal analysis for a distributed coordination model proposed in [8]. One of the first analytically rigorous formulations and treatments of consensus can be found in [9,10]. In the last fifteen years a large number of interesting results covering a variety of consensus aspects have been published. Topics such as distributed optimization and task assignments, coordination in swarms and flock formation, sensor fusion, and distributed estimation and control, have been extensively studied. A large number of references are given in survey papers [6,11–13], as well as in recently published research monographs [14–16]. These references consider a diverse set of issues such as the presence of noise and delay in communication links between agents, time varying topologies, asynchronous

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updating of agent states, quantization effects, and nonlinear dynamics. The following paragraphs provide a brief review of recent approaches to adaptive consensus.

Recent work on adaptive consensus: One of the earlier works on adaptive synchronization in dynamical networks is [17]. The authors assume that the synchronous solution of the overall autonomous network is known and is global information to be tracked by individual agents as a reference trajectory. The local control input is proportional to the error between the agent state and the reference trajectory. The authors prove that this error converges to zero. Similar results are presented in [18]. The assumption that the synchronous solution is known or that it is bounded is restrictive. In [19] the proof of global synchronization uses a circular condition related to adaptively changing coupling gains. This reference requires that a certain matrix dependent on the coupling gains is negative semidefinite at all times $t > 0$. In [20] the problem of steering a group of agents to a predefined reference velocity is considered. The reference velocity is known to the leader. It is also assumed that the reference velocity model is linear with respect to unknown parameters and known base functions are available to each agent. A decentralized adaptive design is proposed by incorporating relative position and relative velocity feedback. In [21,22] the authors consider the problem where every agent has to track a known or estimated leader trajectory. The agent dynamics are linear with respect to both the unknown parameters and the known basis functions. The leader trajectory is global information. The control signals are adaptive with respect to unknown parameters of agent dynamics. The inter-agent coupling parameter is a nonadaptive predefined constant whose value is global information and is the same for all agents. The local input signal resembles a high-frequency gain feedback in decentralized control methods. In [23] the authors analyze the undirected graph topology, and assume that the high frequency gain (parameter multiplying agent input signal) is known, and it is the same for all agents. They proved the interesting result that each agent state converges toward the average of its neighbors' states. In [24] the consensus problem with a general linear model and Lipschitz nonlinear dynamics is considered. The authors analyze an undirected graph and assume that the linear dynamics are known. The proposed consensus protocol can be implemented in a distributed fashion. A continuous time consensus problem of second order systems governed by a directed graph is considered in [25]. The authors show that the error between any two agent positions converges to zero. They also show that in case of absolute velocity damping all velocities converge to zero, while in the case of relative velocity damping the difference between agent velocities converges to zero. In recent work by Chen et al. [26] continuous time adaptive consensus with unknown identical control directions is considered. The authors analyze an undirected graph and show that the difference between agent states tends to zero.

In [27] the authors consider the finite time leader following problem of multi-agent systems whose dynamics is linear with respect to unknown parameters and known basis functions. Similarly as in [21,23] the inter-agent coupling parameters are non-adaptive, pre-calculated and same for each agent. The leader following is achieved in a finite time. In [28] the consensus problem of networked mechanical systems with time-varying delay and jointly connected topologies is considered. Similarly as in [21,22,27] it is assumed that the high-frequency gain is known, and the inter-agent coupling term in the consensus protocol is non-adaptive with a fixed gain whose value is the same for all agents. In [29] the authors investigate the cooperative control of networked agents with unknown control directions. Assuming undirected graph topology they propose interesting Nussbaum type adaptive controller, and showed that all signals are bounded. They also prove that the difference between any two agent states asymptotically goes to zero. Note that this statement does not imply that all agent states have finite limit.

Contribution and organization: Here we consider a network of heterogeneous agents whose dynamics are described by a double integrator discrete time model with input gain of unknown magnitude. Motivated by the evolution of flocks in biology, or the engineering problem of control of formations of unmanned mobile agents, we set out to find an algorithm for each agent to locally tune the inter-agent coupling parameter so that (i) all agent velocities converge to the same value, and (ii) the distance between any two agents converges to a finite limit without using a predefine reference (velocity or position) trajectory.

The proposed algorithm is a normalized gradient recursion based on minimizing the square of the error between an agent state and the one step delayed average of the state's of its neighbors. In the following we list our contribution relative to the recent work of Junmin and Xudong [29]. Ref. [29] considers continuous-time adaptive consensus; we analyze discrete-time adaptive consensus. In [29] the consensus algorithm is constructed based on the Lyapunov function argument while our algorithm is a normalized gradient scheme derived by minimizing a certain quadratic cost function and it is different than the algorithm in [29]. We consider double integrator discrete-time dynamics, while in [29] a single integrator continuous-time system is discussed. Ref. [29] analyzes an undirected graph while we consider a more general directed graph topology. In [29] it is shown that agent states are bounded and the error between any two agents states goes to zero. Note that this statement does not imply that all agent states have a limit. We prove that all agent states converge to the same value. In [29] it is shown that the coupling parameters are bounded, not necessarily convergent functions. We prove that the coupling parameters are convergent sequences. In addition we show that the distance between any two members of the group converges toward a finite limit.

The paper is organized as follows. Section 2 presents the problem formulation. Section 3 presents the proposed algorithm. Analysis of the algorithm is presented in Section 4. A simulation example is given in Section 5. We use the following notation: $\Re$ denotes the set of real numbers; the superscript T denotes the transpose of a matrix; $\rho(A)$ denotes the spectral radius of matrix A; $\|x\|$ is the Euclidean norm of vector x, and $\mathrm{sgn}(a)$ is the sign function of a real number a. Furthermore, ℓ is used to denote a vector with all entries equal to one, *i.e.*, $\ell^T = [1, 1, \dots, 1]$. When performing majorizations and in certain upper bounds, c_i, $i = 1, 2, \dots$ is used to denote nonnegative constants whose values are unimportant.

2. Problem statement

Consider a cooperative group of N agents where the dynamics of the ith agent are described by the following discrete time system

$$x_i(t+1) = x_i(t) + v_i(t) \tag{1}$$

$$v_i(t+1) = v_i(t) + \beta_i u_i(t), \quad i = 1, \dots, N \tag{2}$$

where time $t \geq 0$ takes on nonnegative integer values, $x_i(t) \in \Re$ and $v_i(t) \in \Re$ are the position and velocity respectively, while $u_i(t) \in \Re$ is the control signal or consensus protocol of the agent. In (2) $\beta_i \in \Re$ is an unknown input gain. The model defined by Eq. (2) can be thought of as a discrete time version of a kinematic model

$$\frac{d}{d\tau} v_i(\tau) = \frac{1}{m_i} u_i(\tau), \quad \tau \geq 0,$$

for $i = 1, \dots, N$, where $v_i(\tau)$ is velocity and $u_i(\tau)$ is driving force of the ith agent respectively, while m_i is its mass. Then parameter β_i in (2) can be interpreted as an inverse of m_i. Inspired by