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Adaptive control of passifiable linear systems with quantized measurements and bounded disturbances[☆]

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ABSTRACT

We consider a linear uncertain system with an unknown bounded disturbance under a passification-based adaptive controller with quantized measurements. First, we derive conditions ensuring ultimate boundedness of the system. Then we develop a switching procedure for an adaptive controller with a dynamic quantizer that ensures convergence to a smaller set. The size of the limit set is defined by the disturbance bound. Finally, we demonstrate applicability of the proposed controller to polytopic-type uncertain systems and its efficiency by the example of a yaw angle control of a flying vehicle.

1. Introduction

Adaptive control plays an important role in the real world problems, where exact system parameters are often unknown. One of the possible methods for adaptive control synthesis is the *passification method* [1]. Starting from the works [2,3] this method proved to be very efficient and useful. Nevertheless, while implementing passification-based adaptive control, several issues may arise. First of all, disturbances inherent in most systems can cause infinite growth of the control gain. This issue may be overcome by introducing the so-called "σ-modification" [4,5]. Secondly, the measurements can experience time-varying unknown delay. This problem has been recently studied in [6]. In this paper we consider passification-based adaptive control in the presence of measurement quantization and propose a switching procedure for the controller parameters that ensures the convergence of the system state to an ellipsoid whose size depends on the upper bound of the disturbance.

Control with limited information has attracted growing interest in the control research community lately [7–10]. Due to limited sensing capabilities, defects of sensors and limited communication channel capacities it is reasonable to assume that only approximate value of the output is available to a controller. These sensor and communication imposed constraints can be modeled by quantization [11].

Although adaptive control of uncertain systems received considerable interest and has been widely investigated, there are few works devoted to adaptive control with quantized measurements. In [12] the performance of an adaptive observer-based chaotic synchronization system under information constrains has been analyzed. A binary coder–decoder scheme has been proposed and studied in [13] for synchronization of passifiable Lurie systems via limited-capacity communication channel. In [14] a direct adaptive control framework for systems with *input* quantizers has been developed. In [15] a supervisory control scheme for uncertain systems with quantized measurements has been proposed. In supervisory control schemes usually a finite family of candidate controllers is employed together with an estimator-based switching logic to select the active controller at every time.

Differently from these works, the control scheme proposed here does not require any estimator or observer. Unlike [15] we consider adaptive tuning of the controller gain, rather than switching between several known controllers. At the same time, to ensure convergence to a smaller set, our controller switches parameters of the adaptation law.

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Notations. By $\|\cdot\|$ we denote Euclidean norm for vectors and spectral norm for matrices. For $P \in \mathbb{R}^{n\times n}$ notation $P > 0$ means that P is symmetric and positive-definite, $\lambda_{\max}(P)$, $\lambda_{\min}(P)$ are the maximum and minimum eigenvalues, respectively, P^T denotes transposed matrix P.

2. System description

Consider an uncertain linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t), \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

with state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}$, output $y \in \mathbb{R}^l$, and constant uncertain matrices A, B, C of appropriate dimensions. Unknown disturbance $w(t) \in \mathbb{R}^n$ has a bounded norm:

$$\|w(t)\| \le \Delta_w, \quad t \ge 0.$$

Following [1] we introduce the notion of *hyper-minimum-phase* (HMP) systems.

Definition 1. For a given $g \in \mathbb{R}^l$ the transfer function $g^TW(s) = g^TC(sI - A)^{-1}B$ is called hyper-minimum-phase (HMP) if $g^TW(s)\det(sI - A)$ is a Hurwitz polynomial with a positive leading coefficient $g^TCB > 0$.

Assumption 1. There exists $g \in \mathbb{R}^l$ such that $\|g\| = 1$ and the transfer function $g^TW(s) = g^TC(sI - A)^{-1}B$ is HMP.

The condition $\|g\| = 1$ is imposed only to simplify calculations and is not restrictive since if $g^TW(s)$ is HMP then $\|g\|^{-1}g^TW(s)$ is also HMP.

Remark 1. The search of the vector g satisfying Assumption 1 in general is a difficult problem. It is equivalent to the search of a Hurwitz polynomial in an affine family of polynomials which is probably NP-hard (cannot be solved in a polynomial time, see [16]). One approach based on Monte-Carlo method can be found in [17].

2.1. Passification lemma

Our results are based on the following lemma [3,18].

Lemma 1 (*Passification Lemma*). *The rational function $g^TW(s) = g^TC(sI - A)^{-1}B$ is HMP if and only if there exist a matrix P, a vector $\theta_* \in \mathbb{R}^l$, and a scalar $\varepsilon > 0$ such that*

$$P > 0, \qquad P\bar{A} + \bar{A}^TP < -\varepsilon P, \qquad PB = C^Tg, \tag{2}$$

where $\bar{A} = A - B\theta_*^TC$.

Remark 2. If $g^TW(s) = g^TC(sI - A)^{-1}B$ is HMP then there exists θ such that the input $u = -\theta^Ty + v$ makes the system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned}$$

strictly passive with respect to a new input v, i.e. there exist functions $V(x) = x^TPx$, with $P > 0$, and $\varphi(x) \ge 0$, where $\varphi(x) > 0$ for $x \ne 0$, such that

$$V(x(t)) \le V(x(0)) + \int_0^t \left[y^T(s)gv(s) - \varphi(x(s))\right] ds.$$

Remark 3. Passification lemma is also contained in [19] (implicitly) and in [20] (explicitly). This lemma provides conditions for existence of an output static feedback $u = -\theta^Ty$ that renders the closed-loop system strictly positive real (SPR). If no such constant output feedback exists, then no dynamic output feedback with a proper transfer matrix exists to make the closed-loop system SPR [21]. More subtle results for the case of non-strict passivity can be found in [22].

2.2. Quantizer model

Further we will assume that the controller receives quantized measurements. Following [7] we introduce a *quantizer with a quantization range M and a quantization error bound Δ_e* as a mapping $q: y \mapsto q(y)$ from $\mathbb{R}^l$ to a finite subset of $\mathbb{R}^l$ such that

$$\|y\| \le M \Rightarrow \|q(y) - y\| \le \Delta_e.$$

We will refer to the quantity $e = q(y) - y$ as the *quantization error*. The concrete codomain of q is not important for our further analysis, therefore, can be chosen arbitrary. The value of M is usually dictated by the effective range of a sensor.

By *dynamic quantizer* we will mean the mapping

$$q_\mu(y) = \mu q\left(\frac{y}{\mu}\right), \tag{3}$$

where $\mu > 0$. For each positive μ one obtains a quantizer with the quantization range μM and the quantization error bound $\mu\Delta_e$. We can think of μ as the "zoom" variable: increasing μ corresponds to zooming out and essentially obtaining a new quantizer with larger quantization range and quantization error bound, whereas decreasing μ corresponds to zooming in and obtaining a quantizer with a smaller quantization range but also a smaller quantization error bound. A useful example to keep in mind is a camera with optical zooming capability: one can zoom in and out while the number of photodiodes in the image sensor is fixed. Another example is the system with digital communication channel that can transmit a finite number of bytes. In this case one needs to encode all possible values of the output signal to transmit it through a communication channel. Obviously, in such case one can reduce the quantization error by reducing the range.

3. Ultimate boundedness

Together with the system (1) that satisfies Assumption 1 with some g we consider the adaptive controller

$$\begin{aligned} u(t) &= -\theta^T(t)q(y(t)), \\ \dot{\theta}(t) &= \gamma q(y(t))q^T(y(t))g - a\theta(t), \end{aligned} \tag{4}$$

where $\gamma > 0$ is a controller gain parameter and $a > 0$ is a regularizing parameter. Since $q(y(t))$ is piece-wise continuous we consider right-hand side derivative. As it has been previously shown [23] adaptive controllers similar to (4) without quantization ($q(y) = y$) can ensure ultimate boundedness of the system (1). Here we analyze this controller in the case of quantized measurements.

We will derive our results using the following Lyapunov function

$$V(x, \theta) = x^TPx + \gamma^{-1}\|\theta - \theta_*\|^2, \tag{5}$$

where P, θ_* satisfy (2). For convenience define the following quantities:

$$\Lambda_C = \|C\|, \qquad \lambda_P = \lambda_{\min}(P), \qquad \Lambda_P = \lambda_{\max}(P). \tag{6}$$

Remark 4. Since chattering on the boundaries between the quantization regions is possible, solutions to differential equation (1), (4) are to be interpreted in the sense of Filippov. However, this issue will not play a significant role in the subsequent stability analysis. Indeed, all upper bounds on $\dot{V}$ that we will establish remain valid (almost everywhere) along Filippov's solutions (cf. [24]).