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Output regulation of nonlinear output feedback systems with exponential parameter convergence[☆]

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ABSTRACT

This paper revisits the global robust output regulation (GROR) problem of nonlinear output feedback systems with uncertain exosystems by error output feedback control. The problem was conventionally tackled by employing a linear canonical internal model and as a result, suitable adaptive stabilization has to be done for the augmented system to achieve output regulation. Distinguished from that, a novel nonlinear internal model approach is developed in the present study that successfully converts the GROR problem into a *robust non-adaptive stabilization* problem for the augmented system. The feature of the new approach is two-fold. On one hand, stabilization of augmented system is disentangled from any extra adaptive control law and thus the procedure is simplified. On the other hand, it leads to explicit strict Lyapunov characterization for the closed-loop system and consequently assures exponential parameter convergence.

1. Introduction

One of the celebrated normal-form nonlinear systems is the output feedback one. It has drawn considerable nonlinear control research interest in the last two decades. One may refer to [1,2] and reference therein for a brief overview on early developments. Specifically, relevant stabilization as well as general output regulation problems have been extensively studied; to name but a few, refer to [1–4] for stabilization studies and [5–10] for output regulation ones. Among the aforementioned results, [6,10] studied global output regulation with known exosystems and [5] further considered interesting scenarios on nonlinear internal models. For the same problem with uncertain exosystems, [8,11] studied global adaptive output feedback control. One may refer to [8] for a solid grasp of the so-called global adaptive internal model approach and parameter convergence analysis. It is worth noting that, such internal model approach was first studied for a global disturbance rejection problem with unknown disturbance frequencies in [12] and lately for semi-global output regulation in [13]. Thanks to a properly designed internal model, an output regulation problem can be converted to a stabilization problem for an augmented system. One common feature of existing results in the literature is that the unknown parameters of exosystem are merged to the augmented system and hence managed by certain adaptive stabilization techniques. We shall refer to [14] for an overview of up-to-date results in this direction.

The central idea of the present study is to seek favorable novel construction of internal models serving global output regulation design. Specifically, a convergent estimator for unknown exosystem parameters is incorporated in the internal model dynamics. In this way, we are able to disentangle the augmented system stabilization from integrating any extra adaptive control law and hence the stabilization procedure can be simplified. In particular, the latter non-adaptive stabilization problem can be solved using a ready technique recently developed in [15], since the proposed internal model fully satisfies the required stabilization conditions therein. Moreover, the novel internal model also provides effective estimation of unknown exosystem parameters representing unknown frequencies of exogenous signals. Distinguished from the conventional approaches for similar problems developed in [8,16,17], the proposed non-adaptive output regulation design benefits us not

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only a simplified stabilization solution but also an affirmative characterization of parameter estimation in terms of an explicit strict Lyapunov function. Note that such strict Lyapunov function design also is constructively derived by applying a Lyapunov function construction technique proposed in [18] and in virtue of a persistency of excitation (PE) property as a standing condition in tracking control and disturbance rejection problems. Hence, it assures exponential parameter convergence or in other words, it leads to exponential estimation of unknown exosystem parameters.

It is noted that the proposed internal model design with a convergent estimator can find its early motivation in system estimation of [19]. Some relevant internal model design techniques were studied in [20] for semi-global output regulation of nonlinear strict-feedback systems, and in our companion paper [21] for global output regulation of general lower triangular systems with uncertain exosystems. By contrast, this paper focuses on the GROR design by error output feedback control.

The rest of this paper is organized as follows. Section 2 presents some preliminaries and technical assumptions. Section 3 gives the main result of this paper. Section 4 gives a couple of illustrative examples. Section 5 concludes the paper. The proofs are put in the Appendix.

Throughout the paper, $\|\cdot\|$ is the usual Euclidean norm; I is an identity matrix of a compatible dimension in the context; $Id : \mathbb{R} \to \mathbb{R}$ is an identity function; and $\mathbb{R}_{\geq 0}$ denotes the set of nonnegative real numbers. A function is said (sufficiently) smooth if it is C^k for sufficiently large integer $k \geq 0$ of particular technical requirements. The function $f : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is said to be positive definite if, $f(x) > 0$ for $x \neq 0$ and $f(0) = 0$. A function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$, *i.e.*, $f \in \mathcal{K}$ if, it is continuous, positive definite and strictly increasing. $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}_\infty$ if, it is of class $\mathcal{K}$ and unbounded. The set of bounded $\mathcal{K}$ functions are denoted by $\mathcal{K}^o$, *i.e.*, $\mathcal{K}^o = \mathcal{K} \setminus \mathcal{K}_\infty$. The function $f : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if, for each fixed $s \geq 0$, $\beta(s,t)$ is continuous and decreases to zero as $t \to +\infty$, and for each fixed $t \geq 0$, $\beta(\cdot, t)$ is of class $\mathcal{K}$. For two continuous and positive definite functions $\kappa_1(s)$ and $\kappa_2(s)$, $\kappa_1 \in \mathcal{O}(\kappa_2)$ means $\limsup_{s\to 0^+} \frac{\kappa_1(s)}{\kappa_2(s)} < \infty$. For a pair of functions $f_1(s)$, $f_2(s)$ of compatible dimensions, $f_1 \circ f_2(s) = f_1(f_2(s))$ denotes function composition. For any column vectors $x_1, \ldots, x_r$, we use $\mathrm{col}(x_1, \ldots, x_r)$ to denote $[x_1^\mathsf{T}, \ldots, x_r^\mathsf{T}]^\mathsf{T}$. For any integer $s > 0$, $\iota(s)$ is used to denote $\frac{s}{2}$ rounded down to the nearest integer, *i.e.*, $\iota(s) = \frac{2s-1+(-1)^s}{4}$.

2. Problem and preliminaries

Consider output feedback nonlinear systems described by

$$\begin{cases} \dot{z} = f(z, y, v, w), \\ \dot{x}_i = x_{i+1} + g_i(z, y, v, w), \quad 1 \leq i \leq n,\ x_{n+1} := u, \\ y = x_1, \\ e = y - q(v, w) \end{cases} \tag{1}$$

with state $(z, x) \in \mathbb{R}^{n_0} \times \mathbb{R}^n$, $x = \mathrm{col}(x_1, \ldots, x_n)$ for integers $n_0 \geq 0$ and $n \geq 1$, control input $u \in \mathbb{R}$, performance output $y \in \mathbb{R}$, measured tracking error $e \in \mathbb{R}$, parameter uncertainty $w \in \mathbb{W} \subset \mathbb{R}^{n_w}$, and exogenous signal $v \in \mathbb{V} \subset \mathbb{R}^{n_v}$ generated by

$$\dot{v} = A_1(\sigma)v, \quad v(0) = v_0 \tag{2}$$

with an uncertain parameter $\sigma \in \mathbb{S} \subset \mathbb{R}^{n_\sigma}$. The functions in (1) are globally defined and sufficiently smooth in their arguments and satisfy $f(0, 0, 0, w) = 0$, $g_i(0, 0, 0, w) = 0$, $1 \leq i \leq n$, $q(0, w) = 0$, $\forall w \in \mathbb{W}$. Denote that

$$\mathbb{D} := \mathbb{V} \times \mathbb{W} \times \mathbb{S}.$$

Assume that all the eigenvalues of $A_1(\sigma)$ are distinct with zero real parts for each $\sigma \in \mathbb{S}$, and $\mathbb{V}$ is invariant for (2). The set $\mathbb{D}$ is supposed to be known and compact.

Regarding the system (1) and (2), the GROR problem undertaken in this paper is defined as follows.

Problem 2.1 (*GROR*)**.** For a given compact set $\mathbb{D}$, find an error output feedback controller such that, for each $(v_0, w, \sigma) \in \mathbb{D}$ and each initial value $(z(0), x(0))$ in their entire spaces, the trajectory of the closed-loop system exists and is bounded over the time interval $[0, \infty)$, and moreover, $e(t)$ decays to zero as $t \to \infty$.

Remark 2.1. Problem 2.1 is formulated as a general feedback control problem, including tracking control, disturbance rejection, and stabilization control as rather special cases. Specifically, when $q(v, w) \equiv 0$, Problem 2.1 refers to a disturbance rejection problem and when $v \equiv 0$, it refers to a global robust stabilization control problem. Viewing the exosystem (2) as a source of references and disturbances, we will pursue a basic investigation of Problem 2.1 for tracking control and/or disturbance rejection of nonlinear systems transformable into the form (1) in the presence of any nontrivial solution $v(t)$ of the exosystem (2).

Regarding the plant dynamics (1) having relative degree $n \geq 2$, it is known that there is an extended form by adding an input-driven filter (see, *e.g.*, [22,23])

$$\dot{\xi}_i = -\lambda_i \xi_i + \xi_{i+1}, \quad \lambda_i > 0,\ 1 \leq i \leq n-1,\ \xi_n := u. \tag{3}$$

To achieve that, let

$$\theta = \bar{L}x - \bar{L}\bar{B}\xi, \qquad \xi := \mathrm{col}(\xi_1, \ldots, \xi_{n-1})$$

where $\bar{B} = [\bar{B}_1, \ldots, \bar{B}_{n-1}]$, $\bar{B}_{n-1} = B$, $\bar{B}_i = (A_0 + \lambda_{i+1}I)\bar{B}_{i+1}$, $0 \leq i \leq n-2$, $\bar{B}_0 := \mathrm{col}(\bar{B}_{01}, \ldots, \bar{B}_{0n})$, and

$$\bar{L} = \left[\begin{array}{c|c} -\bar{B}_{02}\bar{B}_{01}^{-1} & \\ \cdots & I \\ -\bar{B}_{0n}\bar{B}_{01}^{-1} & \end{array}\right], \qquad A_0 = \left[\begin{array}{c|c} 0 & I \\ \hline 0 & 0 \end{array}\right], \qquad B = \left[\begin{array}{c} 0 \\ \hline 1 \end{array}\right]$$

of compatible dimensions. Then, the extended system takes the form

$$\begin{cases} \dot{z} = f(z, y, v, w), \\ \dot{\theta} = A_\lambda \theta + \hat{g}(z, y, v, w), \\ \dot{y} = \xi_1 + h(z, \theta, y, v, w), \\ \dot{\xi}_i = -\lambda_i \xi_i + \xi_{i+1},\ 1 \leq i \leq n-1 \end{cases} \tag{4}$$

where A_λ is Hurwitz with a characteristic polynomial $P(s) = \prod_{i=1}^{n-1}(s + \lambda_i)$ and

$$\hat{g}(z, y, v, w) = \begin{bmatrix} g_2(z, y, v, w) - \dfrac{\bar{B}_{02}}{\bar{B}_{01}} g_1(z, y, v, w) + \left(\dfrac{\bar{B}_{03}}{\bar{B}_{01}} - \dfrac{\bar{B}_{02}^2}{\bar{B}_{01}^2}\right) y \\ \vdots \\ g_n(z, y, v, w) - \dfrac{\bar{B}_{0n}}{\bar{B}_{01}} g_1(z, y, v, w) - \dfrac{\bar{B}_{0n}\bar{B}_{02}}{\bar{B}_{01}^2} y \end{bmatrix},$$

$$h(z, \theta, y, v, w) = \theta_1 + \bar{B}_{02}\bar{B}_{01}^{-1} y + g_1(z, y, v, w).$$

Remark 2.2. In the literature, two methods are available for dynamic extension of an output feedback system into an extended form like (4). One is based on a full-order filter; see [17] for its usage in solving the output regulation problem of the same system (1). The other is by a reduced-order filter as (3); see [22,5,10] for strict output feedback systems (*i.e.*, the vector fields of (1) with linear dependence on z) and [3] for the general case.