Systems & Control Letters 92 (2016) 14–19

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

A bilinear differential forms approach to parametric structured state-space modelling

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ARTICLE INFO

Article history:
Received 9 September 2015
Received in revised form
23 November 2015
Accepted 18 February 2016
Available online 24 March 2016

Dedicated to the memory of Jan C. Willems — teacher, colleague, friend

Keywords:
Bilinear differential forms
Loewner approach
Parametric modelling

ABSTRACT

We use one-variable Loewner techniques to compute polynomial-parametric models for MIMO systems from vector-exponential data gathered at various points in the parameter space. Instrumental in our approach are the connections between vector-exponential modelling via bilinear differential forms and the Loewner framework.

1. Introduction

Parametric system identification arises in those areas where system dynamics depends on one or more parameters, e.g. varying geometric or material properties. One approach for its solution is that of [1,2], based on *two-variable* rational interpolation techniques and a Loewner matrix associated with the data. In the *single-input*, *single-output* case, this approach produces a transfer-function or generalized state-space model depending on one parameter in a higher-order polynomial way.

In this paper we use *one-variable* Loewner techniques to compute input-state-output (i/s/o) polynomial-parametric models on the basis of *MIMO* vector-exponential trajectories produced by a system at various points in the parameter space. Instrumental in our approach are the connections established in [3,4] between vector-exponential modelling via bilinear differential forms and the Loewner framework. The basic tool in this work is the Loewner matrix and its rank-revealing factorizations, from which a set of state trajectories is computed in a straightforward way. Different factorizations correspond to different state trajectories: we show that by suitably factorizing the Loewner matrix one can compute also *structured* polynomial-parametric i/s/o models, and we apply this to the case of *passive systems*.

A few remarks are in order to define the scope of our results. Firstly, no assumptions are made on the parametric dependence of the underlying system, except that at each point in the parameter space where data has been collected the system can be described by a set of linear, constant-coefficient differential equations. Secondly, our choice of model class as that consisting of i/s/o linear, time-invariant models that depend polynomially on a parameter is dictated by purely pragmatic reasons, and does not reflect any intrinsic belief in the nature of the actual dependence on the parameter. Lacking any special insight in the physics of the system it is not reasonable to assume a priori any specific functional dependency on the parameter; moreover, if such detailed physical knowledge is available, there are more suitable approaches than a representation-free one. Finally, it is well-known (see [5,6]) that functional dependency is not preserved across different representations: for example, an i/o description depending polynomially on a parameter in general does not correspond to an i/s/o polynomially-parametric representation, and *vice versa*. Our choice of polynomially-dependent parametric i/s/o models is thus motivated purely by practical reasons, namely to identify a "simple" *unfalsified* (in the sense of [7]) model for the data.

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http://dx.doi.org/10.1016/j.sysconle.2016.02.014
0167-6911/

The paper is organized as follows: in Section 2 we state the problem, and in Section 3 we state the assumptions standing in the rest of the paper. Section 4 contains the main results and is divided in five subsections, dealing with various aspects of our approach. In Section 5 we apply our results to the parametric identification of passive systems. In the last section of this paper we discuss our results and their limitations, together with some research directions currently being pursued.

We will be using extensively notions from behavioural system theory, bilinear/quadratic differential forms and the Loewner framework; for a thorough exposition we refer to [8–10], respectively.

Notation

The space of n dimensional real (complex) vectors is denoted by $\mathbb{R}^n$ (respectively $\mathbb{C}^n$), and that of $m \times n$ real matrices by $\mathbb{R}^{m\times n}$. $\mathbb{R}^{\bullet\times m}$ denotes the space of real matrices with m columns and an unspecified finite number of rows. Given matrices $A, B \in \mathbb{R}^{\bullet\times m}$, $\mathrm{col}(A, B)$ denotes the matrix obtained by stacking A over B. The ring of polynomials with real coefficients in the indeterminate s is denoted by $\mathbb{R}[s]$; the ring of two-variable polynomials with real coefficients in the indeterminates ζ and η is denoted by $\mathbb{R}[\zeta, \eta]$. $\mathbb{R}^{r\times q}[s]$ denotes the set of all $r \times q$ matrices with entries in s, and $\mathbb{R}^{n\times m}[\zeta, \eta]$ that of $n \times m$ polynomial matrices in ζ and η. The set of rational $m \times n$ matrices is denoted by $\mathbb{R}^{m\times n}(s)$.

The set of infinitely differentiable functions from $\mathbb{R}$ to $\mathbb{R}^q$ is denoted by $\mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^q)$. $\mathfrak{D}(\mathbb{R}, \mathbb{R}^q)$ is the subset of $\mathfrak{C}^\infty(\mathbb{R}, \mathbb{R}^q)$ consisting of compact support functions. Given $\lambda \in \mathbb{C}$, we denote by $e^{\lambda\cdot}$ the exponential function whose value at t is $e^{\lambda t}$.

2. Problem statement

We assume that at point p in the parameter space the generating system is controllable, represented in *observable image form* as

$$w = M_p\left(\frac{d}{dt}\right)\ell, \tag{1}$$

where $M_p \in \mathbb{R}^{\mathtt{w}\times\mathtt{m}}[s]$; we also assume that $w = \begin{bmatrix} u \\ y \end{bmatrix}$ with u input and y output variables. The input–output partition of the external variables corresponds to a partition

$$M_p(s) =: \begin{bmatrix} U_p(s) \\ Y_p(s) \end{bmatrix}, \tag{2}$$

where $U_p \in \mathbb{R}^{\mathtt{m}\times\mathtt{m}}[s]$ is nonsingular, and $Y_p \in \mathbb{R}^{\mathtt{p}\times\mathtt{m}}[s]$.

The data are vector-exponential trajectories at various frequencies and values of the parameter p, namely

$$w_{p_i,\lambda_{i,j}}(t) = \overline{w_{p_i,\lambda_{i,j}}}e^{\lambda_{i,j}t}, \quad i = 1, \ldots, N',\ j = 1, \ldots, N \tag{3}$$

where $\lambda_{i,j} \in \mathbb{C}$ and $\overline{w_{p_i,\lambda_{i,j}}} \in \mathbb{C}^{\mathtt{w}}$, $i = 1, \ldots, N'$, $j = 1, \ldots, N$. Since (1) is observable, for every $\overline{w_{p_i,\lambda_{i,j}}}$ corresponding to the external trajectory $w_{p_i,\lambda_{i,j}}(\cdot)$ there exists a unique vector $\overline{s_{p_i,\lambda_{i,j}}} \in \mathbb{C}^{\mathtt{m}}$ such that

$$\overline{w_{p_i,\lambda_{i,j}}} = M_{p_i}(\lambda_{i,j})\overline{s_{p_i,\lambda_{i,j}}}. \tag{4}$$

Under some assumptions stated in the next section, we want to compute from the data (3) a parametric state-space model

$$\begin{aligned} \frac{d}{dt}x &= A(p)x + B(p)u \\ y &= C(p)x + D(p)u, \end{aligned} \tag{5}$$

where $A(p) \in \mathbb{R}^{n\times n}[p]$, $B(p) \in \mathbb{R}^{n\times\mathtt{m}}[p]$, $C(p) \in \mathbb{R}^{n\times\mathtt{p}}[p]$ and $D(p) \in \mathbb{R}^{\mathtt{m}\times\mathtt{m}}[p]$, with the property that $(A(p_i), B(p_i), C(p_i), D(p_i))$ defines an *unfalsified state-space model* for the data (3); that is, for all $i = 1, \ldots, N'$ and $k = 1, \ldots, N$ there exists a state trajectory $x = x_{i,k}$ satisfying (4) with $\mathrm{col}(u, y) = w_{p_i,\lambda_{i,k}}$. Such a model (4) will be called an *unfalsified parametric i/s/o model* for the data (3). A refinement of such problem consists in requiring also that the transfer function $C(p)(sI_n - A(p))^{-1}B(p) + D(p)$ is positive-real for all values of p.

In the following we use also a *kernel representation* of (1):

$$R_p\left(\frac{d}{dt}\right)w = 0, \tag{6}$$

where $R_p \in \mathbb{R}^{\mathtt{p}\times\mathtt{w}}[s]$ represents the dynamics at the point p in the parameter space. The i/o partition (2) is reflected in the following partition of R:

$$R_p(s) =: \begin{bmatrix} Q_p(s) & -P_p(s) \end{bmatrix}, \tag{7}$$

where $P_p \in \mathbb{R}^{\mathtt{p}\times\mathtt{p}}[s]$ is nonsingular, and $Q_p \in \mathbb{R}^{\mathtt{p}\times\mathtt{m}}[s]$.

3. Assumptions

The standing assumptions in the rest of this paper are the following:

1. For each p_i, $i = 1, \ldots, N'$, the first $\mathtt{m}$ components of the external variable w are input variables;
2. The transfer function corresponding to each such i/o partition is proper;
3. For each p_i, $i = 1, \ldots, N'$, the McMillan degree of (1) is n.

With reference to (2) and (7), and using standard behavioural system theory, it is straightforward to verify that assumptions (1)–(3) are equivalent with

1′. For $i = 1, \ldots, N'$, $U_{p_i}(s)$ and $P_{p_i}(s)$ are nonsingular;
2′. $Y_{p_i}(s)U_{p_i}(s)^{-1} = P_{p_i}(s)^{-1}Q_{p_i}(s)$ is proper, $i = 1, \ldots, N'$;
3′. $\deg\left(U_{p_i}(s)\right) = \deg\left(P_{p_i}(s)\right) = n$ for $i = 1, \ldots, N'$.

We moreover assume that

4. For $i = 1, \ldots, N'$, the data (3) is *sufficiently informative*, in the sense that an unfalsified state-space model for the data at point p_i

$$\begin{aligned} \frac{d}{dt}x &= A_{p_i}x + B_{p_i}u \\ y &= C_{p_i}x + D_{p_i}u, \end{aligned} \tag{8}$$

can be computed from it.

Several different conditions on (3) guarantee that assumption (4) is satisfied; for example, it can be shown that if the following conditions are satisfied:

4′. $N > n(n + \mathtt{p} + \mathtt{m})$;
4″. for $i = 1, \ldots, N'$, $\lambda_{i,j} \neq \lambda_{i,k}$ for $j \neq k$,

then a *unique* model can be computed.

4. Parametric state-space modelling

4.1. Overview

Our approach is based on the following idea: we first compute from the *primal data* (3) a set of *dual data*, i.e. of vector-exponential trajectories generated by the dual system at the value p_i of the parameter; crucial in such first step is the concept of *mirroring*.

Subsequently, for each value of the parameter p_i we generate from the primal and dual data a *Loewner matrix* $\mathbb{L}_{p_i} \in \mathbb{C}^{N\times N}$, which we proceed to factorize in a rank-revealing way. From