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Global output-feedback stabilization for a class of uncertain time-varying nonlinear systems☆

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ABSTRACT

This paper investigates the global output-feedback stabilization for a class of uncertain time-varying nonlinear systems. The remarkable structure of the systems is the presence of uncertain control coefficients and unmeasured states dependent growth whose rate is inherently time-varying and of unknown polynomial-of-output, and consequently the systems have heavy nonlinearities, serious uncertainties/unknowns and serious time-variations. This forces us to explore a time-varying plus adaptive methodology to realize the task of output-feedback stabilization, rather than a purely adaptive one. Detailedly, based on a time-varying observer and transformation, an output-feedback controller is designed by skillfully combining adaptive technique, time-varying technique and well-known backstepping method. It is shown that, with the appropriate choice of the design parameters/functions, all the signals of the closed-loop system are bounded, and furthermore, the original system states globally converge to zero. It is worth mentioning that, the heavy nonlinearities are compensated by an updating law, while the serious unknowns and time-variations are compensated by a time-varying function. The designed controller is still valid when the system has an additive input disturbance which, essentially different from those studied previously, may not be periodic or bounded by any known constant.

1. Introduction

During the past decades, much effort has been devoted to the global output-feedback control for nonlinear systems [1–13], since it requires less information on the systems and hence is more practical. As a key component of output-feedback control, an observer is needed to reconstruct the unmeasured system states. However, observer theory is far from well-developed, which makes it very hard to find an appropriate observer for some specific questions. Moreover, heavy nonlinearity and coupling in the systems may result in huge obstacles in the design and analysis of output-feedback control. Therefore, there are many unsolved and challenging problems on this active research field, which are of either practical or academic importance and worthy of study.

In recent years, global output-feedback control design has drawn intense attention for nonlinear systems with unmeasured states dependent growth [5,7,10–18]. Specifically, in [5], the output-feedback stabilizing controller was designed for a class of nonlinear systems with known constant growth rate, and the case of unknown constant growth rate was solved in [10,14]. Subsequently, [7] and [15] investigated the cases of known and unknown polynomial-of-output growth rate, respectively. Works [11,16] and [17] considered the case with uncertain control coefficient, by introducing high-gain K-filters and high-gain observers, respectively. It is worth pointing out that the methods in [5,7,10,11,14–17] are merely applicable to the systems with time-invariant growth. Towards this limitation, we proposed time-varying methods in [12,13,18], to successfully design output-feedback controllers for the nonlinear systems with serious uncertainties/unknowns. Detailedly, in [12], the global output-feedback stabilization was solved for a class of nonlinear systems with unknown control coefficients and unknown time-varying growth and in [13], the stochastic case was considered. In [18], the systems investigated are of so-called high-order type and allow time-varying control coefficients.

This paper is devoted to the global stabilization via time-varying output-feedback for a class of nonlinear systems with uncertain control coefficients and unmeasured states dependent growth whose rate is inherently time-varying and of unknown polynomial-of-output. Due to the presence of serious time-variations, the control design schemes in the closely related

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literature [15,16] are inapplicable for the systems under investigation. Moreover, the heavy nonlinearities make the control design methods in [12,13,18] invalid. To deal with the heavy nonlinearities and the serious uncertainties/unknowns and time-variations, an adaptive plus time-varying output-feedback control design scheme is proposed. Specifically, a transformation is introduced to lump uncertainties in control coefficients together and the transformed system is derived. By designing a time-varying observer to reconstruct the unmeasured states of the transformed system, and by introducing a time-scaling transformation, the so-called entire system is obtained, from which, an output-feedback controller is designed by skillfully combining adaptive technique, time-varying technique and backstepping method. By appropriately choosing the design parameters/functions, for any initial condition, all the signals of the closed-loop system are bounded, and furthermore, the original system states, the observer states and the control input ultimately converge to zero. Moreover, we explore the capacity of disturbance compensation of the designed controller. Remark that the additive input disturbance is bounded by an unknown constant. Since the disturbance is not necessarily periodic, it is essentially different from those studied previously (see, e.g., [19–21]) and no method has been found to its asymptotic compensation so far. It is shown that the designed controller is still valid for the case with the disturbance and can achieve the same control objective as the above case without disturbance, except the controller not converging to zero.

The main contributions of this paper are threefold: (i) *The presence of serious time-variations makes this paper essentially different from the closely related literature* [15,16]. In present paper, the systems under investigation admit serious time-variations since the time-varying argument in the system growth can be an arbitrary nonnegative continuous function of time. This is remarkably different from the existing literature [15,16] where weak time-variations are allowed since the system growth is a time-invariant function of system states. Notably, the growth rate in present paper is of polynomial-of-output and of arbitrary nonnegative time-varying function. The former argument shows the nonlinearities of the system heavier than those in [12] although where more serious unknowns in control coefficients are admitted. The latter argument renders the time-variations more serious than those in [18] where the system growth rate is of polynomial-of-time, although where the system is of high-order type. (ii) *A time-varying observer and an updating law are introduced to effectively reconstruct the unmeasured system states and compensate the heavy system nonlinearities, respectively.* Due to the existence of serious time-variations in the systems under investigation, the filters/observers in the closely related literature [15,16] do not work anymore. Moreover, the time-varying observers introduced in [12,18] are invalid for dealing with the polynomial-of-output argument in the system growth. In present paper, we introduce another time-varying observer, in which a time-varying design function and an updating law are introduced to dominate and compensate the serious unknowns and time-variations, and the heavy system nonlinearities, respectively. Furthermore, by skillfully combining adaptive technique and time-varying technique, we propose the design scheme of time-varying output-feedback controller. (iii) *The designed controller can compensate rather general additive input disturbance.* The disturbance in this paper is bounded by an unknown constant, and hence it is considerably different from those studied previously, see, e.g., [19–21] where the disturbances are all periodic and the methods therein are not applicable any more.

The remainder of the paper is organized as follows. Section 2 formulates the control problem. Section 3 introduces a time-varying observer based on which the design scheme is proposed for output-feedback controller. Section 4 summarizes the main results of the paper. Section 5 gives a simulation example and Section 6 provides some concluding remarks.

The following notations will be used throughout this paper. Let $\mathbf{R}$, $\mathbf{R}_+$, $\mathbf{R}_{\geq t_0}$ and $\mathbf{R}^n$ denote the set of all real numbers, the set of all nonnegative real numbers, the set of all real numbers not less than t_0 and the real n-dimensional space, respectively. For any $a \in \mathbf{R}$, $\mathsf{sign}(a)$ denotes its sign function, that is, $\mathsf{sign}(a) = 1$ when $a > 0$, $\mathsf{sign}(a) = 0$ when $a = 0$, and $\mathsf{sign}(a) = -1$ when $a < 0$. For any vector or matrix X, X^{T} denotes its transpose, and $\|X\|_1$ and $\|X\|$ denote its 1-norm and 2-norm, respectively. Clearly, $\frac{1}{\sqrt{n}}\|X\| \leq \|X\|_1 \leq \sqrt{n}\|X\|$ where n is the dimension of vector X. For any column vector $x \in \mathbf{R}^n$, x_i denotes its ith element, and $x_{[i]} = [x_1, \ldots, x_i]^{\mathrm{T}}$. For constants β_i, $i = 1, \ldots, n$, we let $\beta_{i\sim j} = \prod_{k=i}^{j} \beta_k$, $1 \leq i \leq j \leq n$. Moreover, I_n and 0_n denote the n-dimensional identity matrix and the n-dimensional zero column vector, respectively; $\mathbf{diag}[a_1, \ldots, a_n]$ denotes the n-dimensional square matrix whose diagonal elements are a_i's and others are zero.

2. Problem formulation

We consider the global output-feedback stabilization for the following uncertain time-varying nonlinear system:

$$\begin{cases} \dot{\eta}_i = g_i\eta_{i+1} + \phi_i(t, \eta, u), & i = 1, \ldots, n-1, \\ \dot{\eta}_n = g_n u + \phi_n(t, \eta, u), \\ y = \eta_1, \end{cases} \tag{1}$$

where $\eta = [\eta_1, \ldots, \eta_n]^{\mathrm{T}} \in \mathbf{R}^n$ is the system state vector with the initial value $\eta(t_0) = \eta_0$; $u \in \mathbf{R}$ and $y \in \mathbf{R}$ are the control input and output, respectively; g_i, $i = 1, \ldots, n$ are uncertain nonzero constants, called *control coefficients*; $\phi_i : \mathbf{R}_{\geq t_0} \times \mathbf{R}^n \times \mathbf{R} \to \mathbf{R}$, $i = 1, \ldots, n$ are unknown, continuous in the first argument and locally Lipschitz in the rest ones. Throughout this paper, we suppose only the system output y is measurable and make the following assumptions:

Assumption 1. The signs of g_i, $i = 1, \ldots, n$ are known, and there exist known positive constants $\underline{g}_i$, $\overline{g}_i$, $i = 1, \ldots, n$, such that

$$\underline{g}_i \leq |g_i| \leq \overline{g}_i, \quad i = 1, \ldots, n.$$

Assumption 2. There exist a known smooth function $h : \mathbf{R}_{\geq t_0} \to \mathbf{R}_+ \backslash \{0\}$ which satisfies:

$$\begin{cases} \dot{h}(t) \geq 0, \quad h(t_0) > 0, \\ \lim\limits_{t\to+\infty} h(t) = +\infty, \\ \lim\limits_{t\to+\infty} \dfrac{\dot{h}(t)}{h^2(t)} = 0, \end{cases} \tag{2}$$

a known positive integer p and an unknown positive constant θ, such that for any $t \in \mathbf{R}_{\geq t_0}$, $\eta \in \mathbf{R}^n$ and $u \in \mathbf{R}$,

$$|\phi_i(t, \eta, u)| \leq \theta h(t)\big(1 + |y|^p\big)\|\eta_{[i]}\|_1, \quad i = 1, \ldots, n. \tag{3}$$

Assumption 1 shows that the control coefficients of system (1) are unknown but belong to a known compact set. Assumption 2 means that system (1) has unmeasured states dependent growth whose rate is inherently time-varying and of unknown polynomial-of-output. Moreover, Assumption 2 implies that the origin (i.e., $x = 0$) is the equilibrium point of system (1). It is worth noting that Assumption 2 makes system (1) essentially different from those studied in [15,16] whose system growth rates are time-invariant, and hence the control design methods in [15,16] are invalid for system (1).