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Control strategy for state and input observer design

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ABSTRACT

In this note, an alternative to classical methods for observer design is discussed, based on a dual control approach: it is indeed highlighted how an *observer* (at least approximate) can be obtained for a system *by designing a control law for an auxiliary copy of this system*, so that it tracks the system output. An important ingredient in this approach is the use of *high-gain* in the control design. The strategy is illustrated by various examples, including the case of unknown input reconstruction.

1. Introduction

Various approaches are continuously being developed for the challenging problem of state estimation in dynamical systems, based on Lyapunov techniques as in high-gain designs following [1], on the related notion of dissipativity [2], on finite-time convergence as in sliding-mode or homogeneous approaches [3,4], on optimization [5], etc., most of them being directed to a target of *state reconstruction*. In the present paper, the idea is to address this problem as a *control* one, extending the approach formerly developed in this way for wind speed estimation in wind turbines [6]. A first advantage is to make it possible to tackle the problem of *input estimation* in the same fashion as that of state estimation, a topic which has been less investigated, and often in special cases (see for instance [7] for an early result on linear systems, [8] for a class of uncertain systems taking advantage of results on Unknown Input Observers, and later extended in [9], or [10] for a case of periodic input estimation, [11,12] for sliding-mode approaches for nonlinear systems in some canonical forms, or [13] for a detailed identifiability analysis). A second advantage is to inherit methodologies available for control design.

In order to better specify the proposed point of view, let us recall that state observers are often motivated by a problem of control design, in the sense that they are designed for a control purpose, giving rise to *observer-based feedback control*. In the present paper, the main point is to discuss some kind of a dual point of view, namely how a controller can conversely be designed for an observer purpose, leading to a paradigm of *control-based observer design*. In that case, it is interesting to point out that, as a dual result of stabilization by observer-based feedback control – which was shown to be possible for a quite large class of systems by means of *high-gain observer* techniques, and in a *semiglobal* sense [14,15] – one here obtains that a control-based observer design may in general be achieved by means of *high-gain control*, and in some rather *practical* sense (the observation error can be made to reach any arbitrarily small neighbourhood of zero by appropriate tuning).

The main idea in short is that given a nonlinear system described by a state-space representation as follows:

$$\dot{x} = f(x, u); \qquad y = h(x) \tag{1}$$

where x classically denotes the state vector, u the control input, and y the measured output, and satisfying some appropriate observability property, if one can drive an observer system of the form:

$$\dot{\hat{x}} = F(\hat{x}, u, \hat{v}); \qquad \hat{y} = h(\hat{x}) \tag{2}$$

with F such that $F(\hat{x}, u, 0) = f(\hat{x}, u)$, and some control input $\hat{v}$ such that $\hat{y}$ reproduces y, and approaches zero in that case, then $\hat{x}$ provides an estimate of x.

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From that point of view, the observer problem clearly becomes a (tracking) *control problem*. This is actually a way to obtain the well-known Kalman observer equations in the case of linear systems (see for instance [16]). In the present note, this idea is investigated for more general (nonlinear) cases.

Notice that, as said above, this approach can also be used for *input reconstruction*, if one considers that $\hat{v}$ in (2) is looked for so that $\hat{y}$ tracks y output of:

$$\dot{x} = F(x, u, v); \qquad y = h(x) \tag{3}$$

where v is some unknown input ($v = 0$ in the case of system (1)).

Clearly here, with this approach, v can be estimated by $\hat{v}$, provided again that 'enough observability' is satisfied.

The remainder of the paper goes as follows: Section 2 formally states the main ideas of this control-based paradigm for observer design. Section 3 then presents some examples of actual design for special classes of systems, and Section 4 proposes some illustrative simulation results. Section 5 finally concludes the paper.

2. Control-based strategy for observer design

Let us consider a system of general form (1) with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and f, h smooth functions.

Let us assume that $u(.)$ is a given (smooth) function.

Let us finally denote by $x_u(t, x_{t_0})$ the solution at time t of the state equation in (1), under input $u(t)$ over time interval $[t_0, t]$, satisfying $x_u(t_0, x_{t_0}) = x_{t_0}$.

Then, for this input function, an appropriate property for a possible observer design can be expressed in a quite general form as follows (see e.g. [17]):

$$\exists T, \alpha > 0 : \forall x_t, x'_t,$$
$$\int_t^{t+T} \|h(x_u(\tau, x_t)) - h(x_u(\tau, x'_t))\|^2 d\tau \geq \alpha \|x_t - x'_t\|^2;$$
$$t \geq t_0 \tag{4}$$

(this for instance reduces to the standard Kalman *uniform complete observability* in the case of linear time-varying systems.)

The main idea for control-based observer design can be summarized as follows:

Proposition 2.1. *Assume that system (1) satisfies (4) for its given input function $u(t)$, and that one can find a system (2) such that:*

(i) $F(x, u, 0) = f(x, u)$ *for any* x, u;
(ii) *Given* $t_0 \geq 0, \forall \varepsilon > 0, \forall t_1 > t_0, \exists \hat{v} = k(\hat{x}, t) : \forall t \geq t_1, \forall \hat{x}_{t_0}, x_{t_0}, \|h(\hat{x}_u(t, \hat{x}_{t_0})) - h(x_u(t, x_{t_0}))\| \leq \varepsilon$; *and* $\|k(\hat{x}_u(t, \hat{x}_{t_0}), t)\| \leq \varepsilon$;
(iii) $\exists \gamma_1, \gamma_2, \gamma_3 \in \mathbb{R}^{+\star} : \forall x, u, v, \left\|\frac{\partial F}{\partial x}\right\| \leq \gamma_1, \left\|\frac{\partial F}{\partial v}\right\| \leq \gamma_2, \left\|\frac{\partial h}{\partial x}\right\| \leq \gamma_3$.

Then, for any $\varepsilon_2 > 0$, and any $t_2 > t_0$, there exists $\hat{v} = k_2(\hat{x}, t)$ such that:

$$\forall t \geq t_2, \forall \hat{x}_{t_0}, x_{t_0}, \quad \|\hat{x}_u(t, \hat{x}_{t_0}) - x_u(t, x_{t_0})\| \leq \varepsilon_2. \quad \blacksquare$$

In short, this means that if one can control some kind of a copy of the system under study, so as to make it track the measured output with an arbitrarily fast rate, an arbitrary accuracy, and an arbitrarily small control, then one can get from this copy state estimates arbitrarily close to the system states. This turns the observer problem into a control one.

Proof. Given some u, some initial time t_0, and some initial conditions for (1) and (2), let in short $x, \hat{x}$ respectively denote the solutions of (1), (2), and given any $t \geq t_0$, let $\hat{x}_0(\tau)$ denote the solution of (2) for $\tau \geq t$, when $\hat{v}(\tau) = 0$, $\forall \tau \geq t$ and $\hat{x}_0(t) = \hat{x}(t)$. Then:

$$\begin{aligned}
&\|\hat{x}(t) - x(t)\|^2 \\
&\quad = \|\hat{x}_0(t) - x(t)\|^2 \\
&\quad \leq \frac{1}{\alpha} \int_t^{t+T} \|h(\hat{x}_0(\tau)) - h(x(\tau))\|^2 d\tau, \quad \text{by (4) and (i)} \\
&\quad \leq \frac{1}{\alpha} \int_t^{t+T} \|h(\hat{x}_0(\tau)) - h(\hat{x}(\tau))\|^2 d\tau + \frac{\varepsilon^2 T}{\alpha}, \\
&\qquad \text{by (ii) on } h \text{ (for } t \geq t_1) \\
&\quad \leq \frac{\gamma_3{}^2}{\alpha} \int_t^{t+T} \|\hat{x}_0(\tau) - \hat{x}(\tau)\|^2 d\tau + \frac{\varepsilon^2 T}{\alpha}, \quad \text{by (iii) on } h.
\end{aligned}$$

At this stage, notice that again by using condition (iii), as well as (ii) on k, one can check that over $[t, t + T]$ (still for $t \geq t_1$):

$$\|\hat{x}_0(\tau) - \hat{x}(\tau)\| \leq \beta(\gamma_1, \gamma_2, T)\varepsilon$$

for some $\beta > 0$ only depending on γ_1, γ_2, T. Hence:

$$\|\hat{x}(t) - x(t)\|^2 \leq \frac{\gamma_3^2 \beta^2 \varepsilon^2}{\alpha} + \frac{\varepsilon^2 T}{\alpha} \quad \text{for } t \geq t_1.$$

Finally, for any ε_2 (and any t_2), one can choose ε (and t_1)—*i.e.* $\hat{v}$, small enough so that $\|\hat{x}(t) - x(t)\| \leq \varepsilon_2$ for $t \geq t_2$, which ends the proof. $\diamond$

The approach of Proposition 2.1 quite naturally extends to the case of a system (1) subject to some unknown input as in (3). If indeed the observability of (3) extends to that of the system in $(x^T \; v^T)^T$, and if one can drive an auxiliary system (2) with some appropriate $\hat{v}$ so that its output tracks the output of system (3), it is likely that $\hat{v}$ approaches v (this in fact requires that $\dot{\hat{v}}$ be close to $\dot{v}$, as in Proposition 2.1). Obviously those results provide *formal* statements for the idea of control-based observer design, but are not directly constructive, in the sense that they bring the observer problem to a tracking control problem, which should be robust w.r.t. the unknown variations of the reference output. Their purpose is to settle the main ideas, and some actual examples of explicit solutions are proposed in next sections.

3. Some constructive examples and discussion

Let us first consider the case of a linear system (3) as:

$$\dot{x} = Ax + Bu + Dv; \qquad y = Cx \tag{5}$$

and assume the following:

(a1) The pair (A, C) is observable;
(a2) System (6) is controllable (with control v):

$$\dot{x}_0 = Cx; \qquad \dot{x} = Ax + Dv \tag{6}$$

(a3) $x^{(k)}(\cdot)$ is uniformly bounded for $k = 0$ to n, and $v^{(k)}(\cdot)$ for $k = 0$ to $2n - 1$ (by some V)

where (a1) is consistent with the purpose of state estimation, (a2) with the control-based approach, and (a3) just means that only bounded signals are considered. Then, we can state the following:

Proposition 3.1. *For system (5) satisfying* (a1)–(a3), *any* $\varepsilon > 0$, $t_1 > 0$, *one can find gains* F, f_0 *such that for* $\hat{v}$ *as:*

$$\begin{aligned}
&\hat{v} = -F\hat{x} - f_0\hat{x}_0 \\
&\dot{\hat{x}}_0 = C\hat{x} - y; \qquad \dot{\hat{x}} = A\hat{x} + Bu + D\hat{v}
\end{aligned} \tag{7}$$

any $t \geq t_1$, *and* $k = 0, \ldots, n - 1$:

$$\|C\hat{x}^{(k)}(t) - y^{(k)}(t)\| \leq \varepsilon, \qquad |\hat{v}(t) - v(t)| \leq \varepsilon + \sum_{i=1}^{m} \frac{b_i}{b_0} V \tag{8}$$