Systems & Control Letters 74 (2014) 90–97

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Second-order leader-following consensus based on time and event hybrid-driven control ☆

Mengtao Cao [a], Feng Xiao [b], Long Wang [a,*]

[a] Center for Systems and Control, College of Engineering, Peking University, Beijing, 100871, China

[b] Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin, 150080, China

ARTICLE INFO

Article history:
Received 19 January 2014
Received in revised form
27 May 2014
Accepted 20 August 2014
Available online 26 September 2014

Keywords:
Multi-agent systems
Leader-following consensus
Edge events
Hybrid-driven control

ABSTRACT

The leader-following consensus problem of multi-agent systems with double-integrator dynamics is considered in this paper. Herein, there is only one leader, the interaction topology among the followers is undirected, and the followers are reachable from the leader. A novel consensus protocol based on time and edge event hybrid-driven techniques is proposed, and two associated event-triggering rules are presented. Each edge event relies on the information of the corresponding two neighboring agents and event-triggering actions over different edges are independent of each other. It is shown that the proposed protocol can solve the leader-following consensus problem. Moreover, the controller-updating costs and communication costs are largely reduced. Finally, simulations are given to demonstrate the effectiveness of our theoretical results.

1. Introduction

In the last decade, multi-agent coordination has been intensively studied [1–9], and many results have been obtained and applied in many areas, such as formation control [10], flocking [11,12], and complex networks [13].

Consensus problem is an important and challenging research topic in multi-agent coordination, which is to design a local control protocol for each agent so that all agents can share the same state. In [14], Jadbabaie et al. successfully answered the consensus problem raised in the Vicsek Model [15]; in [1], Olfati-Saber et al. extended the result given in [14] and designed consensus protocols for continuous-time systems. And then many researchers devoted themselves to developing consensus theory. Xie and Wang considered a consensus problem under fixed and switching topologies in [4]. Xiao et al. considered a finite-time control problem for multi-agent systems in [10]. Zheng et al. considered a consensus problem of heterogeneous multi-agent systems in [7]. For the case of leader-following topologies, Hong et al. considered a leader-following consensus problem for a multi-agent system with a switching topology, and designed distributed observers in [16]. Ni and Cheng studied the leader-following consensus problem of higher-order multi-agent systems under fixed and switching topologies, and designed distributed controllers to solve the leader-following consensus problem [17]. In [18], Zhu and Cheng considered the leader-following consensus problem of multi-agent systems with double-integrator dynamics, and gave a necessary and sufficient condition for the case of fixed topologies and a sufficient condition for the case of switching topologies under the assumption that the total period, over which all followers were reachable from leader, was sufficiently large.

Most existing results on sampled-data consensus are based on periodic data-samplings [19,20]. Periodic sampled-data control considers the behavior of systems at sampling instants to ensure consensus but ignores the inter-sample behavior, and moreover, in most cases, there is no need for every agent to communicate with its neighbors at each of sampling instants. Event-driven control is an alternative approach to scheduling data-samplings. It is a new topic in multi-agent systems in recent years, and only few results have been obtained [9,21,22]. In [21], Dimarogonas et al. constructed an event-based protocol for solving first-order consensus problems based on the ratio of a certain measurement error with respect to the norm of a function of states, and then these control laws were revised by a self-triggered approach to avoid keeping track of the state errors. In [22], Xiao et al. studied the average consensus problem of first-order integrators

☆ This work is supported by the National Science Foundation of China (NSFC, Grant Nos. 61020106005, 61375120, 61422302 and 61273030), and the Program for New Century Excellent Talents in University (NCET-13-0178).

* Corresponding author.
E-mail addresses: mtcao@pku.edu.cn (M. Cao), fengxiao@hit.edu.cn (F. Xiao), longwang@pku.edu.cn (L. Wang).

http://dx.doi.org/10.1016/j.sysconle.2014.08.013
0167-6911/

with undirected information links, and proposed two kinds of consensus protocols, which were based on asynchronous data-sampling and periodical event detections. However, in event-driven consensus control, the considered systems usually require continuous examining of event-triggering conditions and continuous data exchanges, and sometimes, they may exhibit Zeno behavior. With this motivation, we aim to propose a method, which combines time-driven sampled-data control and event-driven control together, to drive the considered multi-agent systems to reach a consensus on states at reduced controller-updating costs and communication costs.

In this paper, we continue with our previous work [23] and present a new sampled-data consensus protocol for the multi-agent systems with double-integrator dynamics and leader-following topologies; particularly, we focus on the multi-agent systems with only one leader. We model the interaction topology among followers by an undirected graph, and information links from the leader to followers are unidirectional. Each edge of the topology represents a communication link between the two associated agents. We define edge events on each edge, and their occurrences activate the state sampling and controller update of the two associated agents. In our protocol, the communication over each link is triggered independently of those over other links. Our main contribution is the presentation and validity analysis of a consensus protocol for multi-agent systems with double-integrator dynamics and two associated time and edge event hybrid-driven rules to ensure state consensus. These rules also have the advantage of reduced controller-updating costs and communication costs.

This paper is organized as follows. The problem is formulated in Section 2. Consensus analysis is presented in Section 3. Two kinds of event-triggering rules are studied in Section 4. Simulations are given to demonstrate the effectiveness of our theoretical results in Section 5. Finally, conclusions are stated in Section 6.

Notations. *Let* $\mathbf{0} = [0, 0, \dots, 0]^T \in \mathbb{R}^n$, $\mathbf{1} = [1, 1, \dots, 1]^T \in \mathbb{R}^n$, *and* $\|W\| = \sqrt{\lambda_{\max}(W^TW)}$, *where* $\lambda_{\max}(W^TW)$ *is the largest eigenvalue of matrix* W^TW.

2. Problem formulation

Consider a multi-agent system with $n + 1$ double integrators. Label these agents with 0 through n, where the agent, labeled with 0, plays the role of the leader, and the others are referred to as followers. The dynamics of the leader is described as follows:

$$\begin{cases} \dot{x}_0(t) = v_0(t) \\ \dot{v}_0(t) = 0, \end{cases} \tag{1}$$

and the dynamics of the followers is

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \end{cases} \quad i = 1, 2, \dots, n, \tag{2}$$

where $x_i(t), v_i(t) \in \mathbb{R}$ are the position and velocity of agent i respectively. $u_i(t)$ is the state feedback, called protocol, to be designed based on the positions and the velocities of its neighbors and its own.

The leader-following consensus problem of system (1)–(2) is said to be solved if for the designed protocol and any initial states,

$$x_i(t) \to x_0(t), \qquad v_i(t) \to v_0(t), \quad \text{as } t \to \infty,\ \forall i = 1, 2, \dots, n.$$

The interaction topology among followers is modeled by an undirected graph $\mathcal{G}_f = (\mathcal{V}_f, \mathcal{E}_f)$ without multiple edges and self-loops, where $\mathcal{V}_f = \{v^1, v^2, \dots, v^n\}$ and $\mathcal{E}_f \subseteq \{(v^i, v^j) | v^i, v^j \in \mathcal{V}_f, i \neq j\}$. Each element in the vertex set $\mathcal{V}_f$ represents an agent. An edge $(v^i, v^j) \in \mathcal{E}_f$ implies that agents j can receive the state information of agent i. Clearly $(v^i, v^j) \in \mathcal{E}_f$ implies $(v^j, v^i) \in \mathcal{E}_f$. A path is a sequence of distinct vertices $v^{i_1}v^{i_2}\cdots v^{i_m}$, such that for any $k \in \{1, 2, \dots, m-1\}$, edge $(v^{i_k}, v^{i_{k+1}}) \in \mathcal{E}_f$. The topology is connected if for any two vertices v^i and v^j, there exists a path which starts from v^i and ends at v^j. Let v^0 represents the leader. The information links between the leader and followers are unidirectionally from the leader to its followers, and the leader cannot get the information of its followers. Then, we have the overall interaction topology $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, which consists of graph $\mathcal{G}_f$, vertex v^0, and the edges between the leader and its followers. If agent i can receive the state information of agent j, then we say that agent j is a neighbor of agent i. Denote the index set of the neighbors of agent i by $N_i = \{j | (v^j, v^i) \in \mathcal{E}\}, i = 0, 1, \dots, n$. A follower v^i is said to be reachable from the leader if there exists a path which start from v^0 and ends at v^i. The weight matrix $A = [a_{ij}]$ of $\mathcal{G}$ is a matrix with the elements satisfying the property that $a_{ij} > 0$ if and only if $(v^{j-1}, v^{i-1}) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise.

The Laplacian matrix $L = [l_{ij}]$ of $\mathcal{G}$ is defined as follows:

$$l_{ij} = \begin{cases} -a_{ij}, & \text{if } i \neq j, \\ \displaystyle\sum_{k \in N_{i-1}} a_{i(k+1)}, & \text{if } i = j \neq 1. \\ 0, & \text{if } i = j = 1. \end{cases}$$

Let m be the number of edges in $\mathcal{G}$, and label the m edges with 1 through m. For each edge connecting with two followers, we assign an arbitrary orientation, and define the incidence matrix $D = [d_{ij}] \in \mathbb{R}^{(n+1)\times m}$ as follows:

$$d_{ij} = \begin{cases} -1, & \text{if } v^{i-1} \text{ is the tail of the } j\text{th oriented edge}, \\ 1, & \text{if } v^{i-1} \text{ is the head of the } j\text{th oriented edge}, \\ 0, & \text{otherwise}. \end{cases}$$

If let all the edges between the leader and its followers be undirected, then we have a new virtual interaction topology $\widetilde{\mathcal{G}}$, which consists of graph $\mathcal{G}_f$, vertex v^0, and the virtual undirected edges between the leader and its followers. Let $\widetilde{L}$ be the Laplacian matrix of graph $\widetilde{\mathcal{G}}$. Obviously, for the undirected graph $\widetilde{\mathcal{G}}$, the Laplacian matrix and the incidence matrix satisfy that $\widetilde{L} = DWD^T$, where W is the $m \times m$ diagonal matrix, with the weight of the ith edge in the ith diagonal position.

Let $t_0, t_1, t_2, \dots$ be a sequence of time with the property that $t_{s+1} = t_s + h, s = 0, 1, 2, \dots,$ where $h > 0$ is the event-detecting period. At each time of this sequence, each agent decides whether or not to sample the relative state information between itself and its neighbors and update its controller based on the received information. Specifically, if agents i and j are adjacent and their states satisfy some pre-assigned conditions, these two agents sample their relative states and update their controllers; otherwise, their relative state information they used in feedbacks keeps the same value as at the last sampling time. Because all the above events are only relevant to the edge (v^i, v^j), we call them the *edge events* of (v^i, v^j). In the next two sections, we will present the conditions to solve the leader-following consensus problem. Let $t^{ij}_{s'}$ be the time at which the edge event of (v^i, v^j) is activated for the s'th time, and $t_{s^{ij}(t)}$ be the recent time before t at which the edge event of (v^i, v^j) is activated; mathematically,

$$s^{ij}(t) = \max\{s | t_s \in \{t^{ij}_{s'} | t^{ij}_{s'} \leq t\}\}.$$

Clearly, for adjacent followers v^i and v^j, we have $t_{s^{ij}(t)} = t_{s^{ji}(t)}$ for all t.