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Stabilization of persistently excited linear systems by delayed feedback laws

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ABSTRACT

This paper considers the stabilization to the origin of a persistently excited linear system by means of a linear state feedback, where we suppose that the feedback law is not applied instantaneously, but after a certain positive delay (not necessarily constant). The main result is that, under certain spectral hypotheses on the linear system, stabilization by means of a linear delayed feedback is indeed possible, generalizing a previous result already known for non-delayed feedback laws.

1. Introduction

Consider a control system of the form

$$\dot{x}(t) = Ax(t) + \alpha(t)Bu(t), \quad x(t) \in \mathbb{R}^d, \; u(t) \in \mathbb{R}^m, \; \alpha \in \mathcal{G}, \quad (1.1)$$

where x is the state variable, u is a control input, A and B are matrices of appropriate dimensions, and α belongs to a certain class $\mathcal{G}$ of measurable scalar signals $\alpha : \mathbb{R}_+ \to [0, 1]$. This corresponds to the introduction on the linear control system $\dot{x} = Ax + Bu$ of a certain signal α that determines when and how much the control u is active. Note that, when α takes its values on $\{0, 1\}$, (1.1) is actually a switched system between the dynamics of the uncontrolled system $\dot{x} = Ax$ and the controlled one $\dot{x} = Ax + Bu$.

Several different phenomena may be modeled by signal α in (1.1), such as a failure in the transmission of the control u to the plant, a time-varying parameter affecting the control efficiency, or the allocation of control resources, among other possible phenomena. We are interested in general on robust control techniques of (1.1) with respect to α: we suppose that α is not precisely known and we wish our control strategy for (1.1) to be chosen independently of α and to be valid for any signal α in a certain class $\mathcal{G}$.

The problem of controlling (1.1) by a suitable choice of u is obviously not interesting when $\alpha \equiv 0$, or when α is zero for a large amount of time, since in this case the control u has a very limited effect on (1.1). The class $\mathcal{G}$ should thus ensure that the control input has a sufficient amount of action on the system. Among the possible choices for $\mathcal{G}$, the class of (T, μ)-*persistently exciting* signals has attracted much interest recently (see, for instance, [1–8], and also [9] for a similar condition) and, for $T \geq \mu > 0$, it consists on the signals $\alpha \in L^\infty(\mathbb{R}_+, [0, 1])$ such that, for every $t \in \mathbb{R}_+$,

$$\int_t^{t+T} \alpha(s)ds \geq \mu. \quad (1.2)$$

The class of these signals α is noted $\mathcal{G}(T, \mu)$. Further examples of systems similar to (1.1) where the persistent excitation condition appears are given in [5,1,7], where the motivation for the use of persistently exciting signals is also more deeply discussed.

The condition of persistence of excitation (1.2) arises naturally in identification and adaptive control problems (see, e.g., [10–14]). In this context, we are led to study systems of the kind $\dot{x} = -P(t)x$, $x \in \mathbb{R}^d$, where $P(t)$ is a symmetric non-negative definite matrix for every t. If P is bounded and has bounded derivative, it has been shown in [8] that the persistence of excitation of P, in the sense that $\alpha(t) = \xi^T P(t)\xi$ is (T, μ)-persistently exciting for all unitary vectors $\xi \in \mathbb{R}^d$ and for certain constants $T \geq \mu > 0$ independent of ξ, is a necessary and sufficient condition for the global exponential stability of $\dot{x} = -P(t)x$.

We consider the problem of stabilization of system (1.1) to the origin by means of a linear state feedback $u = -Kx$, where we require the choice of the gain matrix K not to depend on a particular signal α but instead on the class $\mathcal{G}(T, \mu)$. In many practical situations, this feedback cannot be done instantaneously, for a

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certain state $x(t)$ may not be available for measure before a certain delay τ has elapsed, and so the state measured in time t is actually $x(t - \tau(t))$. Due to the several practical situations where time lags are introduced by sensors, actuators, or the transmission or processing of signals, the study of delayed systems in general is of much interest, and specially in the context of control systems [15–19]. In several situations, the time-delay appearing in a system is not known exactly and may change with the time, and the literature usually classifies these delays in two types: slowly-varying delays, where its derivative satisfies $|\dot{\tau}(t)| < 1$, and fast-varying delays, without constraints on the derivative of the delay. In this paper, we take as possible delays τ measurable functions taking their values on a certain set $\mathcal{T} \subset \mathbb{R}_+$, and we are thus in the framework of fast-varying delays.

This paper considers the problem of stabilization of (1.1) by a delayed feedback $u(t) = -Kx(t - \tau(t))$, where the delay $\tau(t)$ may depend on t, and the closed-loop system becomes

$$\dot{x}(t) = Ax(t) - \alpha(t)BKx(t - \tau(t)),$$
$$\alpha \in \mathcal{G}(T, \mu),\ \tau \in L^\infty(\mathbb{R}_+, \mathcal{T}) \tag{1.3}$$

where $\mathcal{T} \subset \mathbb{R}_+$ is the set where the delay τ takes its values. The goal of this paper is to present a stabilization result for system (1.3), showing that, under certain hypotheses on A and B, given $T \geq \mu > 0$ and $\tau_0 \geq 0$, there exist a neighborhood $\mathcal{T}$ of τ_0 in $\mathbb{R}_+$ and $K \in \mathcal{M}_{m,d}(\mathbb{R})$ such that, for any $\alpha \in \mathcal{G}(T, \mu)$ and any delay function $\tau \in L^\infty(\mathbb{R}_+, \mathcal{T})$, system (1.3) is exponentially stable, uniformly with respect to α and τ. This generalizes [3, Theorem 3.2], where the same result is given in the case of the non-delayed feedback $u(t) = -Kx(t)$, corresponding thus to $\mathcal{T} = \{0\}$.

Notice that (1.3) is related to switched linear systems with delays, since, when $\alpha(t)$ takes its values on $\{0, 1\}$, (1.3) becomes a switched system between the non-delayed uncontrolled dynamics $\dot{x} = Ax$ and the delayed one $\dot{x}(t) = Ax(t) - BKx(t - \tau(t))$, under the constraint of persistence of excitation given by (1.2). Several results exist concerning switched systems with delays, presented for instance in [20–25]. Many of them apply Lyapunov function and functional techniques to obtain conditions on the systems, the delay and the switching law that guarantee stability under constrained or arbitrary switching, such as [22,24,21]. The constraints on the switching law usually take the form of an average dwell time, as in [25,21,20], or a strategy to design a switching rule, as in [22]. In this paper, we consider that α is an unknown signal satisfying the condition of persistence of excitation (1.2), which is different from the usual hypothesis of average dwell time used for switched systems since α may be active at arbitrarily small time intervals at each time. Our main technique consists on studying (1.3) through a time-contraction procedure and a limit system, which has been proved to be useful when studying persistently exciting systems in [3] but, up to our knowledge, it has not been previously used to study delayed switched systems.

Let us comment briefly on the technique used in [3] to consider the stabilizability of (1.3) in the non-delayed case. The main problem when dealing with the class $\mathcal{G}(T, \mu)$ is that a signal $\alpha \in \mathcal{G}(T, \mu)$ may be zero on certain time intervals, and so the system follows its uncontrolled dynamics $\dot{x} = Ax$. On the other hand, for every $\rho > 0$, it is known by a result from [26] that one can choose a linear feedback $u(t) = -Kx(t)$ that stabilizes (1.1) uniformly with respect to $\alpha \in L^\infty(\mathbb{R}_+, [\rho, 1])$. The main idea in [3] is to perform a change of variables corresponding to a time contraction by a factor $\nu > 0$, which transforms a (T, μ)-signal α into a $(T/\nu, \mu/\nu)$-signal α_ν with $\alpha_\nu(t) = \alpha(\nu t)$. It is possible to show that the family $(\alpha_\nu)_{\nu>0}$ admits a weak-$\star$ convergent subsequence $(\alpha_{\nu_n})_{n\in\mathbb{N}^*}$ in $L^\infty(\mathbb{R}_+, [0, 1])$ with $\nu_n \to +\infty$ and that any weak-$\star$ subsequential limit $\alpha_\star$ of $(\alpha_\nu)_{\nu>0}$ as $\nu \to +\infty$ satisfies $\alpha_\star(t) \geq \mu/T$ almost everywhere. The idea is thus to study a certain limit system obtained as $\nu \to +\infty$, for which stabilization can be obtained using the result from [26] mentioned above. It can then be shown by a limit procedure that the same feedback gain K also stabilizes a time-contracted system for a certain $\nu > 0$ large enough, and one may finally adapt such a feedback gain K in order to obtain a stabilizer for the original system.

This time-contraction technique used in [3] is well-adapted to deal with delays in the feedback, since a delay $\tau(t)$ in the original system will correspond to a delay $\frac{\tau(\nu t)}{\nu}$ in the time-contracted system. We may thus expect to obtain a non-delayed limit system as $\nu \to +\infty$ similar to the one obtained in [3] and to conclude the stabilizability of the original system by a similar argument. This intuition is actually true, as proved in Theorem 2.5, where we prove our stabilizability result by following the same time-contraction argument of the proof of [3, Theorem 3.2].

In their article [3], the authors first prove their stabilization result in the particular case where the dynamics are given by the Jordan block J_d (see (3.1)), since it is a representative example containing most of the difficulties of the proof of the general case. We also treat the case of the Jordan block separately in this article (see Theorem 3.1), but in this particular case we have a stronger result, showing that stabilizability is possible for *any* bounded interval $\mathcal{T} \subset \mathbb{R}_+$ where the delay $\tau \in L^\infty(\mathbb{R}_+, \mathcal{T})$ may take its values, whereas in the general case we may only guarantee stabilizability for delays τ which are perturbations around a certain constant prescribed value τ_0. This difference between the statements of our result in the general case and in the particular case of the Jordan block is more deeply discussed in Section 5.

The plan of the paper is the following. In Section 2, we present the notations and definitions used throughout this paper and recall the previous result of [3]. We then proceed to prove, in Section 3, the main theorem of this paper in the particular case of the Jordan block, which allows us to highlight the main ideas of the proof in a setting where the notations are much clearer than in the general case, and also leads to a stronger result than in the general case. The proof of our main theorem is presented in Sections 4 and 5 discusses the results we obtained, and specially the difference in the statements of Theorems 2.5 and 3.1. The proofs of some technical lemmas used in this paper are given in Appendices A and B.

2. Notations, definitions and previous results

In this paper, $\mathcal{M}_{d,m}(\mathbb{R})$ denotes the set of $d \times m$ matrices with real coefficients, which is denoted simply by $\mathcal{M}_d(\mathbb{R})$ when $d = m$. As usual, we identify column matrices in $\mathcal{M}_{d,1}(\mathbb{R})$ with vectors in $\mathbb{R}^d$. The identity matrix in $\mathcal{M}_d(\mathbb{R})$ is denoted by Id_d and $0_{d\times m} \in \mathcal{M}_{d,m}(\mathbb{R})$ denotes the matrix whose entries are all zero, the dimensions being possibly omitted if they are implicit. The block-diagonal matrix whose diagonal blocks are the square matrices $a_1, \ldots, a_d$ is denoted by $\mathrm{diag}(a_1, \ldots, a_d)$. The notation $\|x\|$ indicates both the Euclidean norm of a vector $x \in \mathbb{R}^d$ and the associated matrix norm. The real and imaginary parts of a complex number z are denoted by $\Re(z)$ and $\Im(z)$ respectively. The sets $\mathbb{R}_+$ and $\mathbb{N}^*$ denote, respectively, the sets of the non-negative real numbers $\mathbb{R}_+ = [0, +\infty)$ and the positive integers $\mathbb{N}^* = \{1, 2, 3, 4, \ldots, \}$. For two topological spaces X and Y, we denote by $\mathcal{C}^0(X, Y)$ the set of all continuous functions from X to Y.

Throughout this paper, we consider the system

$$\dot{x}(t) = Ax(t) + \alpha(t)Bu(t), \quad x(t) \in \mathbb{R}^d,$$
$$u(t) \in \mathbb{R}^m,\ \alpha \in \mathcal{G}(T, \mu), \tag{2.1}$$

where $A \in \mathcal{M}_d(\mathbb{R})$, $B \in \mathcal{M}_{d,m}(\mathbb{R})$, and we take persistently exciting signals α in the class $\mathcal{G}(T, \mu)$ defined as follows.

Definition 2.1. Let T, μ be two positive constants with $T \geq \mu$. We say that a measurable function $\alpha : \mathbb{R}_+ \to [0, 1]$ is a (T, μ)-*signal* if, for every $t \in \mathbb{R}_+$, one has

$$\int_t^{t+T} \alpha(s)ds \geq \mu.$$