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Finite-time observers for multi-agent systems without velocity measurements and with input saturations

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ABSTRACT

This paper is devoted to the distributed finite-time observers for multi-agent systems, where the control inputs are required to be bounded and the velocities are assumed to be not available for feedback. An effective framework through defining a class of coordinated saturation functions is introduced, under which both a first-order finite-time observer and a high-order finite-time observer are constructed. By applying the homogeneous theory for stability analysis, it is proven that all the states of the followers can converge to that of the leader in finite time under our proposed observers. With mild modifications of our control strategies, the foregoing results are then extended to the distributed finite-time containment control problem, where the states of the followers converge to the convex hull spanned by the multiple dynamic leaders. Numerical simulations are presented to demonstrate the efficiency of our methods.

1. Introduction

Distributed control of multi-agent systems has attracted considerable attention due to its multifunctional performance, great efficiency and cost reduction in industrial applications [1–3]. A basic issue arising from distributed control of multi-agent systems is to design protocols such that the agents reach an agreement using local information exchange [4–7]. In [8], the leader-following consensus problem for a group of agents with identical linear systems subject to control input saturation was studied. The global consensus problem for discrete-time multi-agent systems with input saturation constraints under fixed undirected topologies was considered in [9]. In [10], the distributed containment control problem for a group of autonomous vehicles using only position measurements was considered. In [11], distributed containment control problem was considered for multiple autonomous vehicles with double-integrator dynamics. The above distributed control problems are solved asymptotically with infinite setting time. However, convergence rate is an important indicator for the dynamic behaviors of the agents and it is often required that the distributed control problems should be settled within finite time. As a consequence, finite-time control has received great interest in the control community. Compared with conventional asymptotical control or exponential control, finite-time control offers faster response, higher accuracy, and better disturbance rejection and robustness against uncertainties.

Several researchers have achieved preliminary results related to distributed finite-time control for multi-agent systems. In [12], sufficient conditions which guaranteed finite-time convergence were proposed by extending results on nonsmooth stability analysis. In [13], several distributed finite-time consensus rules were constructed for first-order multi-agent dynamics in a unified way. Two classes of finite-time protocols were constructed from the two-dimensional system point of view and were termed as iterative learning protocols in [14]. Distributed finite-time attitude containment control for multiple rigid bodies was addressed for both stationary leaders and dynamic leaders in [15]. In [16], an efficient framework was proposed to achieve finite-time decentralized formation tracking with the introduction of decentralized sliding mode estimators.

With regard to the finite-time control problem, there are still two difficulties to be settled. First, in many practical situations, it is difficult to obtain velocity information or some velocity information cannot be precisely measured. Therefore, the distributed finite-time control algorithms must be designed with only position measurements. Second, in the cooperative control of multi-agent systems, each agent updates its protocol using the interaction information from all its neighbors. However, this control strategy is impractical in reality because the real-life actuators are unable to supply unlimited power. In particular, when the number of agents in a network is very large, it is unavoidable that the information datum from the neighbors exceeds the saturation value of the actuators.

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Motivated by the above mentioned considerations, in this paper we investigate finite-time control by using finite-time observers for double-integrator dynamics subject to input saturations. To the best of our knowledge, there has been no study on this problem because of the lack of effective theoretical tools. In this paper, we first introduce a class of saturation functions, based on which a distributed first-order finite-time observer is constructed. In contrast to the conventional observer-based control, our strategy settles the coordinated control problem within finite time. Then, in order to simplify the control input and make it flexible to adjust the controller gains, we extend the first-order finite-time observer to a high-order finite-time observer. By applying the homogeneous theory for stability analysis, it is proven that all the states of the followers can converge to that of the leader in finite time under both the first-order observer and the high-order observer. Finally, with mild modifications of our control strategies, we extend the foregoing results to the distributed containment control problems, where the states of the followers are driven into the convex hull spanned by those of the multiple leaders. Numerical simulations are presented to demonstrate the efficiency of our proposed protocols.

2. Preliminaries

2.1. Finite-time stability

Definition 1. Consider the system

$$\dot{x} = f(x), \qquad f(0) = 0, \quad x \in \mathbb{R}^n \tag{1}$$

where $f : U_0 \to \mathbb{R}^n$ is continuous in an open neighborhood U_0 of the origin. Let $(r_1, \ldots, r_n) \in \mathbb{R}^n$ with $r_i > 0, i = 1, \ldots, n$ and $f(x) = (f_1(x), \ldots, f_n(x))^T$ be a continuous vector field. Vector function $f(x)$ is said to be homogeneous of degree $\kappa \in \mathbb{R}$ with respect to $(r_1, \ldots, r_n)$ if, for any given $\epsilon > 0$, $f_i(\epsilon^{r_1}x_1, \ldots, \epsilon^{r_n}x_n) = \epsilon^{\kappa+r_i}f_i(x), i = 1, \ldots, n, \forall x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$. System (1) is said to be homogeneous if $f(x)$ is homogeneous.

Lemma 1 ([17]). *Consider the following system:*

$$\dot{x} = f(x) + \hat{f}(x), \qquad f(0) = 0, \quad x \in \mathbb{R}^n \tag{2}$$

where $f(x)$ is a continuous homogeneous vector field of degree $\kappa < 0$ with respect to $(r_1, \ldots, r_n)$, and $\hat{f}$ satisfies $\hat{f}(0) = 0$. Assume $x = 0$ is an asymptotically stable equilibrium of the system $\dot{x} = f(x)$. Then $x = 0$ is a locally finite-time stable equilibrium of system (2) if

$$\lim_{\epsilon \to 0} \frac{\hat{f}_i(\epsilon^{r_1}x_1, \ldots, \epsilon^{r_n}x_n)}{\epsilon^{\kappa+r_i}} = 0, \quad i = 1, \ldots, n, \ \forall x \neq 0. \tag{3}$$

In addition, if system (2) is globally asymptotically stable and locally finite-time stable, then it is globally finite-time stable.

Lemma 2 ([18]). *Consider system (1). Suppose that there are C^1 positive-definite function $V(x)$ defined on a neighborhood of the origin, and real numbers $c > 0$ and $0 < \alpha < 1$, such that $\dot{V}(x) + cV^{\alpha}(x) \leq 0$. Then, the origin is locally finite-time stable. In addition, the settling time, depending on the initial state $x(0) = x_0$, satisfies $T(x_0) \leq \frac{V(x_0)^{1-\alpha}}{c(1-\alpha)}$ for all x_0 in the open neighborhood of the origin.*

2.2. Graph theory

Let $\mathcal{G}(\nu, \varepsilon)$ be an undirected graph of order N with the set of nodes $\nu = \{\nu_1, \nu_2, \ldots, \nu_N\}$ and edges $\varepsilon \subseteq \nu \times \nu$. The index set of neighbors of node ν_i is denoted by $\mathcal{N}_i = \{j : (\nu_i, \nu_j) \in \varepsilon\}$. If there is a path between any two nodes of $\mathcal{G}(\nu, \varepsilon)$, then $\mathcal{G}(\nu, \varepsilon)$ is said to be connected. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} \geqslant 0$, where $a_{ij} > 0$ if and only if $(\nu_i, \nu_j) \in \varepsilon$. The Laplacian matrix of graph $\mathcal{G}$ is denoted by $\mathcal{L}_{\mathcal{A}} = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$. For a cooperative tracking problem, we consider another target node ν_{N+1}. The accesses of $\nu_i \in \nu$ to ν_{N+1} are represented by $\mathcal{B} = \mathrm{diag}\{a_{1(N+1)}, \ldots, a_{N(N+1)}\} \in \mathbb{R}^{N\times N}$, where $a_{i(N+1)} > 0$ if ν_{N+1} is a neighbor of ν_i and $a_{i(N+1)} = 0$ otherwise. The information exchange matrix is defined as $\mathcal{K}(\mathcal{A}, \mathcal{B}) \triangleq \mathcal{L}_{\mathcal{A}} + \mathcal{B}$.

3. Problem statement

Consider a group of agents with double-integrator dynamics given by

$$\dot{q}_i = p_i, \qquad \dot{p}_i = u_i, \quad i \in \mathcal{V} := \{1, \ldots, N\} \tag{4}$$

where $q_i \in \mathbb{R}$ and $p_i \in \mathbb{R}$ are the position and velocity of the ith agent, and $u_i \in \mathbb{R}$ is the control input bounded by $\|u_i\|_\infty \leq u_{\max}$. The dynamics of the leader is given by

$$\dot{q}_{N+1} = p_{N+1}, \qquad \dot{p}_{N+1} = u_{N+1} \tag{5}$$

where $u_{N+1} \in \mathbb{R}$ is the given control input which generates the desired target trajectory. For the agents, an undirected graph is used to model the interaction topology. The dynamics of the leader is only observed by a subset of the followers. For the cooperative tracking problem, the following assumptions are assumed to be held.

Assumption 1. The communication topology between the followers is connected.

Assumption 2. There exists at least one follower who has a path to the leader.

The following lemma can be obtained by matrix theory.

Lemma 3 ([19]). *Under Assumptions 1 and 2, matrix $\mathcal{B}$ has at least one positive entry and the information exchange matrix $\mathcal{K}(\mathcal{A}, \mathcal{B}) = \mathcal{L}_{\mathcal{A}} + \mathcal{B}$ is symmetric and positive definite.*

The control objective discussed in this paper is to design distributed control laws without relative velocity measurements and with input saturations such that the followers track the states of the leader in finite time. More specifically, we have the following definition.

Definition 2. We say a local tracking problem is solved, if there exists an open neighborhood $U_0 \subset \mathbb{R}^2$ of the origin such that for any initial states within U_0, $\lim_{t\to\infty} \|q_i - q_{N+1}\| = 0$, $\lim_{t\to\infty} \|p_i - p_{N+1}\| = 0$, $i \in \mathcal{V}$. The global tracking problem is said to be solved if $U_0 = \mathbb{R}^2$.

We say a local finite-time tracking problem is solved, if there exists an open neighborhood $U_0 \subset \mathbb{R}^2$ of the origin and $T_0 \in [0, \infty)$ such that for any initial states within U_0, $\lim_{t\to T_0} \|q_i - q_{N+1}\| = 0$, $\lim_{t\to T_0} \|p_i - p_{N+1}\| = 0$, and $q_i = q_{N+1}$, $p_i = p_{N+1}$ for $t > T_0$, $i \in \mathcal{V}$. The global finite-time tracking problem is said to be solved if $U_0 = \mathbb{R}^2$.

To proceed, we give for any $\varsigma \in \mathbb{R}$ the following saturation function:

$$s_\gamma^\alpha(\varsigma) = \begin{cases} \mathrm{sig}(\varsigma)^\alpha, & |\varsigma| < \gamma; \\ \gamma^\alpha \mathrm{sign}(\varsigma), & |\varsigma| \geq \gamma \end{cases} \tag{6}$$

where α, γ are positive constants and $\mathrm{sig}(\varsigma)^\alpha$ denotes $\mathrm{sign}(\varsigma)|\varsigma|^\alpha$. An example of the saturation function is illustrated in Fig. 1 with $\alpha = 0.5$ and $\gamma = 1$. For $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, we extend the saturation function to vector form $s_\gamma^\alpha(x) = \left(s_\gamma^\alpha(x_1), \ldots, s_\gamma^\alpha(x_n)\right)^T$.

The following lemma can be obtained directly.

Lemma 4. *If we define*

$$S_\gamma^\alpha(\varsigma) = \begin{cases} \dfrac{|\varsigma|^{\alpha+1}}{\alpha+1}, & |\varsigma| < \gamma; \\ \gamma^\alpha|\varsigma| - \dfrac{\alpha\gamma^{\alpha+1}}{\alpha+1}, & |\varsigma| \geq \gamma, \end{cases} \tag{7}$$