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Wave Propagation Characteristics in Thick Conventional and Auxetic Cellular Plates**

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ABSTRACT Based on Mindlin plate models and Kirchhoff plate models, this study was concerned with the wave propagation characteristics in thick conventional and auxetic cellular structures, with the objective to clarify the effects of negative Poisson's ratio, shear factor and orthotropic mechanical properties on the dynamic behaviors of thick plates. Numerical results revealed that the predictions using variable shear factor in Mindlin plate models resulted in high wave frequencies, which were more significant for plates with negative values of Poisson's ratio. The present study can be useful for the design of critical applications by varying the values of Poisson's ratio.

KEY WORDS auxetic cellular structure, plate, frequency, shear factor, negative Poisson's ratio

I. Introduction

Conventionally, when being axially stretched, a solid is compressed in the perpendicular direction. This phenomenon can be well described by the definition of Poisson's ratio: $\nu = -\varepsilon_2/\varepsilon_1$, where ε_1 and ε_2 are the tensile strains in the stretching direction and perpendicular to the stretching direction, respectively. For widely used engineering materials, the sign of Poisson's ratio is normally positive. For the auxetic materials, however, the sign of Poisson's ratio is negative, because such materials feature lateral extension, instead of shrinking, when stretched.

Although having been found, for example, in iron pyrites, pyrolytic graphite, rock with micro cracks, bone tissues and skin[1] the auxetic materials did not attract much attention until Lakes[2] discovered that isotropic auxetic foams could be manufactured from conventional open-cell foams. Since then, auxetic materials made of isotropic polymers and foams have been developed and designed for potential applications such as impact absorption, indentation resistance, fracture toughness, pressure vessel heads and shock absorbers[1]. Therefore, to explore their additional potential applications, knowledge of their mechanisms is crucial.

In recent years, studies have been focused on the application of auxetic materials modeled as plates with counter-intuitive fascinating properties compared with the conventional materials. The first order sandwich plate theory for the free vibration problem of sandwich laminates in cylindrical bending was proposed by Scarpa and Tomlinson[3]. Their numerical results showed that the re-entrant cell cores provided improvements in bending stiffness capabilities. Ruzzene et al.[4] evaluated the propagation of elastic waves in two-dimensional cellular structures, with the focus on the re-entrant honeycombs After

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that, the results of experimental tests on the foams featuring negative Poisson's ratio exhibited superior damping and acoustic properties to conventional foams[5]. Alderson et al.[6, 7] derived the elastic constants of various chiral and anti-chiral honeycombs, and discussed the deformation mechanisms responsible for the auxetic functionality in such honeycombs. The auxetic triangular cores of high-strength were manufactured by Michelis et al.[8], the mechanical behaviors of which were numerically investigated using the finite element method. The shear modulus of the auxetic cores was found superior to that of the existing mass-produced honeycomb cores. In addition, the dynamic behaviors of auxetic plates have also been a topic of interest, with examples ranging from vibration and wave propagation of sandwich plates with auxetic properties[3] to thermoelastic damping in rectangular auxetic plates[9]. Recently, Lim carried out a series of investigations on the effect of negative Poisson's ratio on bending[10], buckling[11] and free vibration[12] of auxetic plates. However, in Lims' work, the cellular materials were modeled as thick plates with isotropic material properties, which is obviously not the case for cellular materials, such as the re-entrant honeycombs[13]. The material properties of re-entrant honeycombs were found to be orthotropic the Poisson's ratio of which could reach much less than −1. Moreover, the calibration for shear factor has been developed for thick plates It was found that the shear factor in Mindlin plate models depended on the effective Poisson's ratio of the plate materials[14]. As a result, one can infer that the orthotropic mechanical properties and Poisson's ratio can affect the wave propagation characteristics in thick auxetic plates. These two aspects are the motivation of the present study

II. Mechanical Properties of Cellular Structures

Featuring the phenomenon of counter-intuitive negative Poisson's ratio, numerous types of cellular structures which exhibit fascinating characteristics have been well documented[1]. The auxetic cellular structures in term of honeycombs with various ranges of angle θ are illustrated in Fig.1, where the Cartesian coordinate system is chosen such that the coordinates (x, y) are along the in-plane directions and z is along the thickness direction with the origin O located at the mid-plane of the plate. The cellular structures are assumed to be made of linear isotropic materials with Young's modulus E, Poisson's ratio ν and mass density ρ_s. For isotropic materials, the shear modulus G is related to E and ν by $G = E/\left[2\left(1+\nu\right)\right]$. Letting θ and t be the angle and thickness of the cell walls, respectively, by varying the geometry of the structures, i.e., angle θ and the length ratios $t/l, h/l$, the topologies may vary (see Fig.1(c)), leading to different effective mechanical properties. For negative values of angle θ, the geometrical constraint requires that $2l\sin\theta + h > 0$ to ensure that the internal cell vertices remain untouched during the deformation[3]. According to Gibson and Ashby[13], the independent mechanical properties of the structures were derived using the conventional mechanics of materials:

$$E_1 = E\left(\frac{t}{l}\right)^3 \frac{\cos\theta}{\left(h/l+\sin\theta\right)\sin^2\theta} \frac{1}{1+\left(2.4+1.5\nu+\cot^2\theta\right)\left(t/l\right)^2} \tag{1}$$

$$\nu_{12} = \frac{\cos^2\theta}{\left(h/l+\sin\theta\right)\sin\theta} \frac{1+\left(1.4+1.5\nu\right)\left(t/l\right)^2}{1+\left(2.4+1.5\nu+\cot^2\theta\right)\left(t/l\right)^2} \tag{2}$$

$$E_2 = E\left(\frac{t}{l}\right)^3 \frac{\left(h/l+\sin\theta\right)}{\cos^3\theta} \frac{1}{1+\left(2.4+1.5\nu+\tan^2\theta+2\frac{h/l}{\cos^2\theta}\right)\left(t/l\right)^2} \tag{3}$$

$$\nu_{21} = \frac{E_2\nu_{12}}{E_1} = \frac{\sin\theta\left(h/l+\sin\theta\right)}{\cos^2\theta} \frac{1+\left(1.4+1.5\nu\right)\left(t/l\right)^2}{1+\left(2.4+1.5\nu+\tan^2\theta+\frac{2h/l}{\cos^2\theta}\right)\left(t/l\right)^2} \tag{4}$$

$$G_{12} = E\frac{\left(t/l\right)^3\left(h/l+\sin\theta\right)}{F\left(h/l\right)^2\cos\theta} \tag{5}$$

with $F = 1+\frac{2h}{l}+\left(\frac{t}{l}\right)^2\left\{\frac{\left(2.4+1.5\nu\right)\left(2+h/l+\sin\theta\right)}{h/l}+\frac{\left(h/l+\sin\theta\right)\left[\left(h/l+\sin\theta\right)\tan^2\theta+\sin\theta\right]}{\left(h/l\right)^2}\right\}$.