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Multi-objective parallel machine scheduling problem with job deterioration and learning effect under fuzzy environment ☆

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A B S T R A C T

The current paper investigates a non-identical parallel machine multi-objective scheduling problem in which both the deterioration and learning effects have been considered. Due to uncertainty of the parameters in real-world systems, processing times and due dates of jobs are represented here with triangular fuzzy numbers. Using the credibility measure, a nonlinear mathematical model is provided based on fuzzy chance-constrained programming (FCCP) with the aim to minimize two objective functions, namely total earliness/tardiness (ET) and maximum completion time of jobs (makespan). Since it is a mixed integer nonlinear mathematical model, there is no guarantee that the solution will obtain a global optimum. Therefore, a multi-objective branch and bound algorithm is provided by introducing an effective lower bound in order to obtain a Pareto optimal front. Computational results show that the algorithm proposed is especially useful to solve large-scale problems.

1. Introduction

Parallel machine scheduling problems are among the most important problems in scheduling literature, since these are not only a factor in most manufacturing environments, but also have been widely used in information systems, i.e. parallel information processing and distributed computing. For real-world problems, the main challenge is the uncertainty of the parameters affecting the problem. Parameters such as processing times and due dates are typically non-deterministic, and fuzzy numbers can be used to reflect such uncertainty and ambiguity. At a workplace, however, there are many factors that influence the parameters of a problem and its basic parameters. In most cases, raw materials waiting in a queue for processing deteriorate over time and this can lead to a complete loss of raw materials. This phenomenon is known as the "*job's deterioration effect*". On manufacturing lines, on the other hand, operators of each machine will improve their abilities and skills as time passes so that they can more easily perform manufacturing processes. This phenomenon is known as the "*learning effect*". Clearly, not all workshop machines are the same—that is, there are different types and brands of machines in any one workplace. Such differences will cause variations in processing speeds of machines to perform jobs.

In addition to the discussion on workshop environments, some objectives are always of interest in production areas, which can be the result of the perspective of a manufacturer, such as minimizing the maximum completion time, as well as the result of the consumer perspective, such as minimizing the total earliness and minimizing the total tardiness. In the literature, two latter objectives are generally regarded as a single objective, named *earliness/tardiness* (ET) that actually represents the concept of *just in time* (JIT) in the manufacturing environment. To date, to the best of our knowledge, no research has examined non-identical parallel machine scheduling problems with deteriorating jobs and learning effect simultaneously in a fuzzy environment.

There have been many studies of the problems of parallel machine scheduling in a deterministic environment without the deterioration and learning effects, such as Unlu and Mason (2010), Baptiste et al. (2012), Bozorgirad and Logendran (2012), Gerstl and Mosheiov (2012), Shen, Wang, and Wang (2013), Rodriguez, Lozano, Blum, and Garcı'a-Martı'nez (2013), Prot, Bellenguez-Morineau, and Lahlou (2013), Sarıçiçek and Çelik (2011) and Vallada and Ruiz (2011). Some papers have been recently published on deterioration effects. Ji and Cheng (2008) and Liu, Zheng, Wang, and Xu (2013) addressed parallel machine scheduling problems with the objective of minimizing the sum of completion times where deterioration function was linear. Ji and Cheng (2009) considered multi-objective

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parallel-machine scheduling problems with a simple linear deteriorating jobs. The objectives were to minimize the makespan, total machine load, and total completion time. Mazdeh, Zaerpour, Zareei, and Hajinezhad (2010) investigated a parallel machine scheduling problem with an aim to minimize total tardiness and machine deterioration costs. A tabu search algorithm was provided to solve this problem. Cheng, Hsu, and Yang (2011) studied the problem of unrelated parallel-machine scheduling with deteriorating maintenance activities. Ruiz-Torres, Paletta, and Pe'rez (2013) proposed a similar problem in which the deterioration rate of jobs was described by a function of the work sequence, and solved it by using the simulated annealing (SA) algorithm. Further, Yang and Yang (2013), Zhang and Luo (2013) and Wang, Zhou, Ji, and Wang (2014) studied parallel machine scheduling problem with the deterioration of jobs under conditions like maintenance, job rejection and non-simultaneous machine availability.

A few articles on scheduling problems consider only the learning effects; for example, Mosheiov (2001) reported a review of classical scheduling problems with a learning effect; Lee, Chuang, and Yeh (2012) examined scheduling problems with uniform parallel machines; and Liu (2013) evaluated both the learning effect and job delivery times, simultaneously. Considering the effects of learning and deterioration simultaneously under a deterministic environment, Toksari and Güner (2009) evaluated identical parallel machine scheduling problem with the aim to minimize the total ET, nonlinear coefficient of deterioration, and common due dates. It was found that the optimal solution to this problem would be V-shaped. Based on the mathematical model, the authors also provided a lower bound to solve large-sized problems.

To date, much research has been devoted to the problems of parallel machine scheduling under deterministic environments; however, a small number of studies have been published for uncertain parameters. Without the effects of learning and deteriorating jobs, Al-Khamis and M'Hallah (2011) developed a two-stage model for the fundamental problem of job-scheduling on parallel machines and solved it. In the first stage, this model obtains the optimal capacity of machines. And in the next stage, it seeks to optimize the number of on-time jobs when job due dates are interpreted in a probabilistic sense. Peng and Liu (2004) examined the problem of parallel-machines scheduling with fuzzy processing times. The authors developed three approaches of modeling stochastic problems, including expected value programming (EVP), chance-constrained programming (CCP) and dependent-chance programming (DCP) in a fuzzy environment by using the credibility measure. Ultimately, a hybrid intelligent algorithm (the combination of genetic algorithms and simulations) was proposed to analyze the results of models. Further, Gharehgozli, Tavakkoli-Moghaddam, and Zaerpour (2009) presented a fuzzy multi-objective mixed integer goal programming model to simultaneously minimize the sum of weighted tardiness and weighted flow time by considering release time and sequence-dependent setup times.

Yeh, Lai, Lee, and Chuang (2013) has recently published a paper on the effect of learning in the context of scheduling on parallel machines with fuzzy processing times. The objective function was to minimize the maximum completion time of jobs. Using the possibility measure for ranking, the authors achieved a deterministic scheduling problem. Then, two meta-heuristic methods—namely, SA and GA—were used to solve this problem.

To our knowledge, no research has been published on scheduling of parallel machines with the deterioration and learning effects simultaneously when processing times and due dates are represented by fuzzy numbers. In the present study, a non-identical parallel machine scheduling problem is to be investigated in a fuzzy environment with the objective of simultaneously minimizing two objective functions: (1) total earliness/tardiness (ET) and (2) the maximum completion time of jobs (makespan). The deterioration and learning effects both have an influence on conditions of the problem and such influence can be applied at processing times of jobs. We assume that both deteriorating jobs and learning effects are a function of position for job processing. The remaining sections are as follows. Section 2 describes the preliminaries necessary for examining the present problem. Section 3 defines the problem. To do this, we first provide a representation of a multi-objective mixed integer nonlinear programming (MOMINLP) model. Then, a crisp model is developed by using the credibility measure and chance-constrained programming (CCP). Through introducing effective lower bounds for the objective functions, Section 4 provides a multi-objective branch and bound algorithm which can generate a Pareto optimal front. Section 5 shows the experimental results, and finally, Section 6 provides conclusions.

2. Preliminaries

Fuzzy theory was first introduced by Zadeh (1965). This section describes requirements related to fuzzy numbers which will be used in next sections. Definitions and characteristics are given below.

Definition 1. If $\tilde{\xi}_1 = (a, b, c)$ and $\tilde{\xi}_2 = (d, e, f)$ are triangular fuzzy numbers, then the addition and subtraction of these numbers will also represent a triangular fuzzy number (Lee, 2006):

$$\tilde{\xi}_1(+)\tilde{\xi}_2 = (a+d, b+e, c+f) \tag{1}$$

$$\tilde{\xi}_1(-)\tilde{\xi}_2 = (a-f, b-e, c-d) \tag{2}$$

Definition 2. If $\tilde{\xi}$ is a fuzzy number and r is a crisp number, then the relationship between three measurements, i.e. possibility, necessity, and credibility for the event $\tilde{\xi} \leqslant r$ is represented as below (Liu & Liu, 2002):

$$Cr\left\{\tilde{\xi} \leqslant r\right\} = \frac{1}{2}\left(Pos\left\{\tilde{\xi} \leqslant r\right\} + Nec\left\{\tilde{\xi} \leqslant r\right\}\right) \tag{3}$$

Proposition 1. *Zhu and Zhang (2009) show that, if $\tilde{\xi}$ is a triangular fuzzy number and $\lambda > 0.5$, then:*

$$Cr\left\{\tilde{\xi} \leqslant r\right\} \geqslant \lambda \Longleftrightarrow r \geqslant (2-2\lambda)\xi^{(2)} + (2\lambda - 1)\xi^{(3)} \tag{4}$$

where $\xi^{(2)}$ and $\xi^{(3)}$ denote the center and right side of the fuzzy triangular number $\tilde{\xi}$, respectively.

3. Problem formulation

3.1. Problem definition

There are N jobs which are available for processing at time zero on M parallel machines with different speeds of V_j. Each job i must be processed on a machine and job preemption is not allowed. Workers of each machine have different capabilities in terms of gaining skills, which is denoted by a negative learning coefficient of β_j. On the other hand it is assumed that each job is deteriorated by different rates of α_i. Processing times and due dates are given as triangular fuzzy numbers. There are two objective functions to be minimized simultaneously—the summation of earliness and tardiness (ET) and makespan ($C_{\max}$). According to standard representation of scheduling problems (Graham, Lawler, Lenstra, & Kan, 1979), the problem is denoted as follows:

$$Qm|LE, D|(ET, C_{\max}) \tag{5}$$