Operations Research for Health Care ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Operations Research for Health Care

journal homepage: www.elsevier.com/locate/orhc

Hospital competition in prices and quality: A variational inequality framework

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HIGHLIGHTS

- A game theory model is developed that captures competition among hospitals for patients for different procedures.
- The hospitals compete in both pries and quality and maximize utility, which includes net revenue and altruism benefit.
- The qualitative analysis and algorithmic approach are based on variational inequality theory.
- A case study based on several major hospitals in Massachusetts demonstrates the applicability of the framework.

ARTICLE INFO

Article history:
Received 1 April 2017
Accepted 5 October 2017
Available online xxxx

Keywords:
Game theory
Hospitals
Competition
Quality
Healthcare
Variational inequalities

ABSTRACT

In this paper, we construct a game theory model to capture competition among hospitals for patients for their medical procedures. The utility functions of the hospitals contain a revenue component and a component due to altruism benefit. The hospitals compete in prices charged to paying patients as well as in the quality levels of their procedures. Both prices and quality levels are subject to lower and upper bounds. We state the governing Nash equilibrium conditions and provide the variational inequality formulation. We establish existence of an equilibrium price and quality pattern and also present a Lagrange analysis of the equilibrium solutions. An algorithm is proposed and then applied to numerical examples comprising a case study focusing on four major hospitals in Massachusetts.

1. Introduction

Hospitals are essential institutions for the provision of health-care to society, providing medical diagnostics, surgeries, treatments, deliveries of babies, and emergency care. They are complex ecosystems, whose existence depends on delivering quality care to their patients. At the same time, hospitals in the United States are under increasing pressure and stresses with many consolidations in the industry, driven, in part, by needs to reduce costs, as well as to be perceived as being value-based (see Commins [1]). In 2015, there were over 100 hospital and health system consolidations in the United States among over 5,500 registered hospitals [2]. Hospitals are also, often, regulated and have been subject to reforms internationally to enhance competition (see Brekke et al. [3]).

Given the importance of competition as a salient feature of hospitals today, there is a large empirical literature on the relationship between quality and hospital competition [4–6]. Other studies have examined the relationships between competition and health care system costs [7], and between competition and patient satisfaction [8] and [9]. The majority of the empirical literature has been on the US experience, with more recent studies focusing on the United Kingdom and other European countries (see, e.g., Kessler and McClellan [10], Kessler and Geppert [11], Cooper et al. [12]). Of course, it is important to quantify quality in this setting. Specifically, as noted by Gravelle et al. [5], although quality is often measured by hospital mortality, they itemize sixteen different measures of hospital quality, with six of the sixteen quality measures based on standardized mortality rates, seven on standardized readmission, revisions, and redo rates, and three constructed from surveys of patients' experiences.

However, the literature on theoretical frameworks for hospital competition is not as advanced and is primarily the purview of economists rather than operations researchers. For example, Gravelle et al. [5] construct a hospital quality competition model under fixed prices, building on the work of Ma and Burgess [13], Gaynor [14], and Brekke et al. [15]. The model in this paper differs in several significant ways; notably, we have competition in both prices and quality and we consider multiple procedures for each hospital. Plus, our prices and quality levels must satisfy lower and upper bounds. Longo et al. [16] present a simple, yet elegant, two hospital model of quality and efficiency competition. Brekke et al.

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https://doi.org/10.1016/j.orhc.2017.10.002

[3] develop a competitive hospital model in quality with regulated prices in a Hotelling [17] framework using a differential game methodology. Rivers and Glover [7] provide an excellent review of competition and healthcare and emphasize the importance of being able to identify and understand the mechanism of competition in this industry in order to provide higher quality of care and patient satisfaction.

Interestingly, a survey on operations research and healthcare (cf. Rais and Viana [18]) does not mention the term *game theory*, although it does acknowledge the seminal contributions of Roth et al. [19] on kidney exchanges, which, as the latter authors remark, resemble some of the "housing" problems considered in the mechanism design literature for indivisible products. In addition, we note the survey of Moretti [20], which reviews recent applications of coalition games in medical research, along with an identification of some open problems.

We believe that a rigorous game theory framework for hospital competition that can handle price and quality regulations in the form of lower and upper bounds and also enables the computation of equilibrium solutions is valuable. Here we construct such a framework, through the use of the theory of variational inequalities, for the formulation of the governing Nash equilibrium conditions, the qualitative analysis, and the computation of the equilibrium quality and price patterns. For background on the methodology of variational inequalities, but applied to supply chain competition in quality, see the book by Nagurney and Li [21].

This paper is organized as follows. In Section 2, we present the hospital competition model, in which the hospitals compete in both prices and quality for patients for the procedures that they offer. The utility function of each hospital consists of a revenue component and also a component associated with altruism benefit since hospitals are decision-makers in healthcare. Each hospital's benefit function captures the total benefit of the patients from receiving treatment at the hospital (see, e.g., Brekke et al. [15]), weighted by a factor reflecting the monetized value of altruism of the hospital. The demands for procedures at different hospitals are elastic and depend on prices charged as well as the quality levels, whereas the costs of different procedures depend on the quality levels. The prices and quality levels are subject to lower and upper bounds, which allow us to capture different regulations, such as minimum quality standards. Also, if, as in the case of a price for a procedure, one sets the lower bound equal to the upper bound, then one has, in effect, a fixed price, which is useful in modeling such pricing schemes that may occur in different country health systems. We define the Nash equilibrium governing the noncooperative game and present the variational inequality formulation. We also prove that an equilibrium solution is guaranteed to exist.

In Section 3, we construct an alternative formulation of the variational inequality through the use of Lagrange multipliers and give an analysis of the marginal utilities of the hospitals when the prices and quality levels of the hospital procedures lie within or at one of the bounds. Such an analysis enables both hospitals as well as policymakers to assess the impacts of loosening or tightening certain regulations. We note that there are several papers that have contributed to the analysis of the behavior of the solutions to a variational inequality, which models equilibrium problems through the use of Lagrange multipliers. For example, in operations research, the papers by Barbagallo et al. [22] and Daniele et al. [23] have done so for the financial equilibrium problem, and the paper by Daniele and Giuffré [24] for the random traffic equilibrium problem. Also, recently, Daniele et al. [25] analyzed a cybersecurity investment supply chain game theory model with nonlinear budget constraints by means of Lagrange multipliers.

In Section 4, we first describe the algorithm that we use in our case study. The case study consists of four hospitals in eastern Massachusetts and three major procedures that they all provide. In the case study we report, for different scenarios, the computed equilibrium prices and quality levels of the hospital procedures, the demand for these procedures, as well as the incurred net revenues and utilities. We conclude the paper with Section 5, where we summarize our results and provide suggestions for future research.

2. The hospital competition model

We now present the hospital competition model consisting of m hospitals with a typical hospital denoted by i and with each being able to carry out n medical procedures with a typical medical procedure denoted by k. Let p_{ik} denote the price charged by hospital i for procedure k. We group the prices associated with hospital i into the vector $p_i \in R^n_+$ and we then group the vectors of prices of all the hospitals into the vector $p \in R^{mn}_+$. In addition, we let Q_{ik} denote the quality associated with hospital i carrying out procedure k. We group the quality levels of hospital i into the vector $Q_i \in R^n_+$ and the quality levels of all hospitals into the vector $Q \in R^{mn}_+$. The strategic variables of each hospital i; $i = 1, \ldots, m$, are its vector of prices charged and its vector of quality levels for the procedures, which, at the equilibrium, are denoted, respectively, by p^*_i and Q^*_i. All vectors are column vectors.

We assume that there are lower and upper bounds on the price charged by hospital i for procedure k, denoted by $\underline{p}_{ik}$ and $\bar{p}_{ik}$, respectively, so that the prices p_{ik}; $i = 1, \ldots, m$, must satisfy the constraints:

$$\underline{p}_{ik} \le p_{ik} \le \bar{p}_{ik}, \quad k = 1, \ldots, n. \tag{1}$$

Observe that, if, because of regulations, there is a fixed price imposed for a hospital i and procedure k then we set: $\underline{p}_{ik} = \bar{p}_{ik}$. This is standard, for example, in England (cf. Gravelle et al. [5]). We assume that patients undergoing the procedures are responsible for the payments, which may come out of pocket, through insurance, and/or a government subsidy.

In addition, there are bounds associated with the quality levels. Regulatory bodies often impose minimum quality standards, which we denote by $\underline{Q}_{ik}$ for $i = 1, \ldots, m$; $k = 1, \ldots, n$, to ensure a minimum level of quality. At the same time, hospitals may be limited by the maximum level of quality that they can achieve for different procedures with $\bar{Q}_{ik}$ representing the maximum for hospital i and procedure k with $i = 1, \ldots, m$; $k = 1, \ldots, n$. Hence, the following constraints must also hold for each i; $i = 1, \ldots, m$:

$$\underline{Q}_{ik} \le Q_{ik} \le \bar{Q}_{ik}, \quad k = 1, \ldots, n. \tag{2}$$

We let K_i denote the feasible set corresponding to hospital i; $i = 1, \ldots, m$, where $K_i \equiv \{(p_i, q_i) | (1) \text{ and } (2) \text{ hold}\}$. These feasible sets are closed and convex.

The demand for procedure k over the time horizon of interest at hospital i, which is denoted by d_{ik}, is given by the function

$$d_{ik} = d_{ik}(Q, p, \alpha_{ik}), \quad i = 1, \ldots, m; k = 1, \ldots, n, \tag{3}$$

where α_{ik} is a vector of demand parameters that capture the location of patients and other hospitals relative to hospital i, patient preferences over distance and quality, and other factors that can influence a patient's choice sets. Gravelle et al. [5] proposed demand parameter vectors in the context of hospital quality competition; here we refine the vectors from the hospital to the hospital-procedure level. Furthermore, we allow for the demand at i for k to depend on the prices of the procedure not only at i but also at the other hospitals as well as on the prices of other procedures. Moreover, the demand functions can also, in general, depend on the quality levels of all procedures at all hospitals, as well as on the vector of additional demand parameters associated with each hospital and procedure. We assume that d_{ik} is increasing in Q_{ik}