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Approximating Total Weighted Completion Time on Identical Parallel Machines with Precedence Constraints and Release Dates

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Abstract

We present a $(2+2\ln 2+\varepsilon)$-approximation algorithm for the classical nonpreemptive scheduling problem to minimize the total weighted completion time of jobs on identical parallel machines subject to release dates and precedence constraints, improving upon the previously best known 4-approximation algorithm from 1998. The result carries over to the more general problem with precedence delays and generalizes a recent result by Li (2017) for the problem without release dates or delays.

Keywords: approximation algorithm, scheduling, precedence constraints, total weighted completion time, time-indexed LP relaxation

1. Introduction

We investigate the problem to schedule jobs on identical parallel machines subject to precedence delays so as to minimize the total weighted completion time. This is denoted by $\mathrm{P}|\mathrm{prec}(d_{jk})|\sum w_jC_j$ in the standard classification scheme of Graham, Lawler, Lenstra, and Rinnooy Kan [1] and generalizes the problem $\mathrm{P}|r_j,\mathrm{prec}|\sum w_jC_j$ of scheduling jobs subject to precedence constraints and release dates.

We are given a set $N=\{1,\dots,n\}$ of jobs and the number m of identical machines. Each job j has a weight $w_j\ge 0$ and a processing time $p_j\ge 0$; it must be processed nonpreemptively for that amount of time on one of the machines. Each machine can process at most one job at a time. A strict partial order $\prec$ on N specifies precedence constraints; for each pair j,k of jobs with $j\prec k$ we are given a nonnegative precedence delay d_{jk}, meaning that job k can only start d_{jk} time units after j's completion. The goal is to find a feasible schedule S of the jobs, minimizing the total weighted completion time $\sum_{j\in N} w_jC_j^{\mathrm{S}}$, where C_j^{S} denotes the time when job j is completed in the schedule S. Precedence delays can be used to model ordinary precedence constraints ($d_{jk}=0$ for all $j\prec k$) and release dates $r_j\ge 0$, specifying for each job j the first point in time when it can be started (by adding a dummy job 0 of length 0 with precedence delays $d_{0j}:=r_j$).

The problem is strongly NP-hard as it encompasses the strongly NP-hard problems $\mathrm{P}||\sum w_jC_j$, $1|r_j|\sum C_j$, $1|\mathrm{prec}|\sum C_j$, and, by introducing a last dummy job, $\mathrm{P}||C_{\max}$ as special cases; see [2, Problems SS13, SS4, SS8]. Even more, the results of Lenstra and Rinnooy Kan [3] and of Hoogeveen, Schuurman, and Woeginger [4] imply that there is no approximation algorithm with performance guarantee better than 4/3, unless P = NP. Under some stronger version of the Unique Games Conjecture, the problem is even hard to approximate within any factor smaller than 2 [5].

The study of approximation algorithms for the problem $\mathrm{P}|r_j,\mathrm{prec}|\sum w_jC_j$ started with a 13-approximation from 1996 [6]. While this approximation algorithm is based on a time-indexed LP relaxation, the subsequent improving algorithms are grounded upon the LP relaxation in completion time variables introduced by Schulz [7], based on work of Queyranne [8]. These include a 7-approximation [9] and a 5.328-approximation [10]. In 1998, Munier, Queyranne, and Schulz [11, 12] developed a 4-approximation algorithm for the more general problem $\mathrm{P}|\mathrm{prec}(d_{jk})|\sum w_jC_j$. If C_j^{LP}, $j\in N$, denote the values of an optimal solution to the LP relaxation in completion time variables, the jobs are scheduled by a job-driven list scheduling procedure in order of LP-midpoints $C_j^{\mathrm{LP}}-p_j/2$.

Recently, Li [13] gave a $(2+2\ln 2+\varepsilon)$-approximation algorithm for the special case $\mathrm{P}|\mathrm{prec}|\sum w_jC_j$, where all precedence delays are 0. This is the first algorithm for this problem since 1996 that is based on the time-indexed LP relaxation. The solution of the relaxation is used to define values C_j^{LP}, and afterwards, the algorithm runs a job-driven list scheduling procedure in order of LP-α-points $C_j^{\mathrm{LP}}-(1-\boldsymbol{\alpha})p_j$, where $\boldsymbol{\alpha}$ is chosen uniformly at random from the interval $(0,1/2]$.

In this paper Li's approach is generalized to the problem $\mathrm{P}|\mathrm{prec}(d_{jk})|\sum w_jC_j$ that was considered by Munier, Queyranne, and Schulz, improving upon the best

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