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Iterated egalitarian compromise solution to bargaining problems and midpoint domination

Emin Karagözoğlu [a,b,*], Elif Özcan Tok [a,c]

[a] Bilkent University, Turkey
[b] CESifo, Munich, Germany
[c] CBRT, Turkey

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ABSTRACT

We introduce a new solution for two-person bargaining problems: the *iterated egalitarian compromise solution*. It is defined by using two prominent bargaining solutions, the *egalitarian solution* (Kalai, 1977) and the *equal-loss solution* (Chun, 1988), in an iterative fashion. While neither of these two solutions satisfy *midpoint domination* – an appealing normative property – we show that the *iterated egalitarian compromise solution* does so.

1. Introduction

In a seminal paper, Nash (1950) [7] introduced the axiomatic treatment of bargaining problems. Over the last seven decades, the axiomatic approach has attracted a considerable attention from researchers studying bargaining (see [6] for an overview). The axiomatic literature on bargaining has been productive in coming up with solution concepts with appealing normative properties. Two prominent solutions of interest for the current paper are the *egalitarian solution* (*E,* for short) due to Kalai (1977) [3] and the *equal loss solution* (*EL*, for short) due to Chun (1988) [2]. As their names suggest, both solutions apply an egalitarian notion of justice in proposing outcomes to bargaining problems. More precisely, for each bargaining problem, *E* proposes the maximum utility profile that gives each agent an equal gain over his disagreement outcome, whereas *EL* proposes the maximum utility profile that gives each agent an equal loss over his ideal outcome (i.e., the best possible outcome for the agent among the outcomes that are individually rational for both).

These two solutions share a common weakness: both of them fail to satisfy a basic yet desirable normative requirement that a solution should assign each agent at least half of his ideal point outcome in all bargaining problems. It can be rephrased as, for any problem, an outcome proposed by a solution should be Pareto superior to the *randomized dictatorship* outcome. This requirement was introduced in [11] and known as *midpoint domination* (*MD*, for short). As Rachmilevitch (2017) [9] points out, midpoint domination has both fairness and efficiency connotations. On one hand, it requires both agents to receive at least *half* of their ideal point outcomes (fairness) and on the other, it requires the proposed outcome to be *Pareto superior* to the midpoint (efficiency). Hence, it is an appealing normative property.

In this paper, we, first, introduce a new solution concept for two-person bargaining problems: *iterated egalitarian compromise solution* (*IEC,* for short). For a problem where *E* and *EL* propose the same outcome, the outcome proposed by *IEC* coincides with theirs. For a problem where *E* and *EL* propose different outcomes, *IEC* proposes a *compromise* in an iterative fashion, by using the proposed outcomes of *E* and *EL* at each iteration step. Hence, the name, iterated egalitarian compromise. Second, we show that *IEC* is well-defined, i.e. for any problem in the domain of two-person bargaining problems we consider, it proposes a unique outcome, defined as the limit of an iterative process. Finally, we show that *IEC* satisfies midpoint domination despite the fact that neither of the solutions it is based on does so, a fact that makes the result a non-trivial one. A recent attempt in a similar direction is [9]. The author proposes a midpoint-robust (i.e., satisfying midpoint domination) version of *E*.

The paper is organized as follows: in Section 2, we introduce the bargaining problem, define the solutions of interest, and the midpoint domination property. In Section 3, we prove that *IEC* is well-defined and it satisfies midpoint domination. Furthermore,

* Correspondence to: Department of Economics, Bilkent University, 06800 Bilkent, Ankara, Turkey.
E-mail address: karagozoglu@bilkent.edu.tr (E. Karagözoğlu).

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we relate *IEC* to another prominent solution concept that has an egalitarian flavor, the Kalai–Smorodinsky solution [4]. Section 4 concludes with final remarks.

2. The model

A simple two-person *bargaining problem is* denoted by $S \subset \mathbb{R}^2$. It satisfies the following properties: it is (i) non-empty, (ii) closed, and bounded from above, (iii) convex, (iv) comprehensive, (v) $S \cap \mathbb{R}^2_{++} \neq \emptyset$, and (vi) it contains the disagreement outcome, $\mathbf{0} \equiv (0, 0)$. The axiomatic properties of the solutions we will use allow us to normalize the *disagreement outcome* to $(0, 0)$. Since we will do that in what follows, we denote the problem by S instead of (S, d). Intuitively, S represents all the utility vectors that can be achieved by the agents. The non-emptiness is to make the problem non-trivial. The closedness of S means that the set of physical agreements is closed and that agents' payoff functions are continuous. The boundedness from above means that the maximum utility an agent can achieve out of an agreement is finite. The convexity assumption means that agents could agree to take a coin-toss between two outcomes and that each agent's payoff from the coin toss is the average of his/her payoffs from these outcomes. Comprehensiveness stipulates that utility is freely disposable down to the disagreement utilities. $S \cap \mathbb{R}^2_{++} \neq \emptyset$ rules out degenerate problems where no agreement can make all agents better off than the disagreement outcome. Finally, $\mathbf{0} \in S$ means that the agents can agree to disagree. We denote the set of all such problems by Σ. For every $S \subset \mathbb{R}^2$, its weak (strong) Pareto optimal set is defined as $WPO(S) \equiv \{y \in S \mid x > y \text{ implies } x \notin S\}(PO(S) \equiv \{y \in S \mid x \gneqq y \text{ implies } x \notin S\})$. Here, we will focus on a subdomain of Σ, denoted by $\widehat{\Sigma}$, whose weak and strong Pareto frontiers coincide (i.e., the bargaining frontier does not have any horizontal or vertical segments). The importance of this assumption will be explained later in the proof of Proposition 1. Finally, a bargaining solution F is a function, which assigns to any bargaining problem S, a unique point in it.

The *egalitarian solution* [3] equalizes agents' gains over their disagreement outcomes. Accordingly, it assigns to each S the point, $E(S)$ with identical (x, y)-coordinates and $E(S)$ is the maximum possible. This corresponds to selecting the intersection point of the Pareto frontier and the 45-degree line drawn from the disagreement point (in our case, the origin). The *equal loss solution* [2] equalizes agents' losses from their ideal point outcomes. Formally, ideal point, introduced by [4], is defined as $a_i(S) \equiv \max\{s_i : s \in S\}$, where $a_i(s)$ denotes agent i's ideal point outcome. Accordingly, the *equal loss solution* assigns to each S, the point $EL(S) = a(S) - (l, l)$, where l is the minimum possible. This corresponds to selecting the point at the intersection of the Pareto frontier and the 45-degree line drawn from the ideal point. Note that for all $S \subset \Sigma$, if $a_1(S) > a_2(S)$, then $EL_1(S) > E_1(S)$ and $E_2(S) > EL_2(S)$, and vice-a-versa.

A solution F satisfies *midpoint domination*, if it proposes an outcome $F(S) \geq mp(S) \equiv \frac{1}{2}a(S)$, for all S. Fig. 1 shows an example, where both E and EL violate *MD*. Note that the bargaining problem in the example is in $\widehat{\Sigma}$.

The *iterated egalitarian compromise solution* (or *IEC*, for short) assigns to each $S \in \widehat{\Sigma}$, the point x, if $E(S) = EL(S) = x$ and assigns the point $y \equiv \cap_{t\in\mathbb{N}} PO(S_t)$, where $S_0 \equiv S$ and the bargaining problem in iteration step t, S_t, for $t \geq 1$ is derived by applying E and EL to S_{t-1} in a way that, the origin (i.e., the disagreement point) of S_t denoted by $o(S_t)$, is $o(S_t) = (\min\{E_1(S_{t-1}), EL_1(S_{t-1})\}, \min\{E_2(S_{t-1}), EL_2(S_{t-1})\})$ and consequently $a(S_t) = (\max\{E_1(S_{t-1}), EL_1(S_{t-1})\}, \max\{E_2(S_{t-1}), EL_2(S_{t-1})\})$.

IEC could be interpreted as a conflict resolution mechanism, which resolves the *conflict* between E and EL in a step-by-step fashion, by using the minimal outcomes in each iteration as starting points and the maximal outcomes as ideals for the bargaining problem in the next step. Fig. 2 shows how *IEC* operates in a problem where E and EL propose different outcomes.

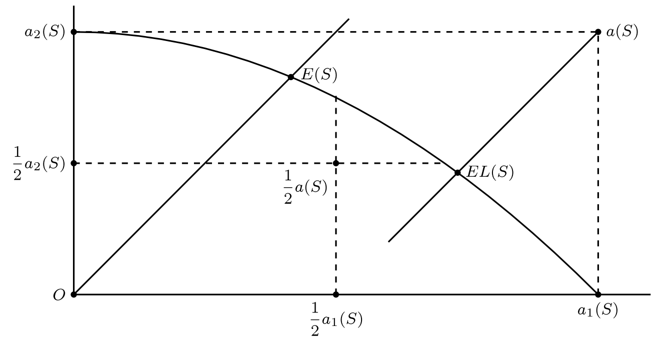

Fig. 1. *E* and *EL* violate *MD*.

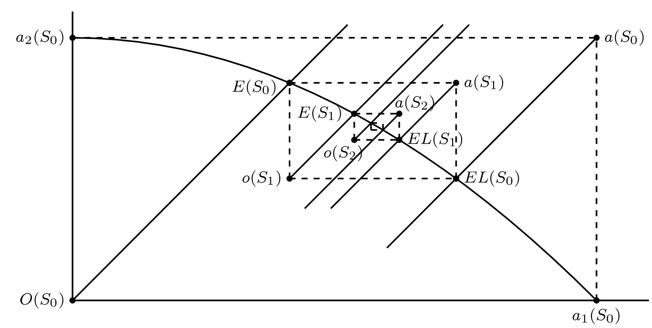

Fig. 2. Iterated egalitarian compromise solution.

3. The result

First, we prove that *IEC* is well-defined, i.e. for all $S \in \widehat{\Sigma}$ the iterative process embedded in *IEC* converges to a single point.

Proposition 1. *For all $S \in \widehat{\Sigma}$,* IEC *is well-defined.*

Proof. First, consider a symmetric bargaining problem, $S \equiv S_0$. In this case, *IEC* proposes a single outcome, since $E(S_0) = EL(S_0)$. Now, consider an asymmetric problem, $S \equiv S_0 \in \widehat{\Sigma}$. Without loss of generality, suppose that $a_1(S_0) > a_2(S_0)$. For notational convenience, let $a_1(S_t) - o_1(S_t) = \alpha_t$ and $a_2(S_t) - o_2(S_t) = \beta_t$. Since both E and EL operate via upward-sloping 45-degree lines, for each iteration step t, we get $\alpha_{t+1} + \beta_{t+1} = |\alpha_t - \beta_t|$. The sequences (α_t) and (β_t) are decreasing and bounded below ($\alpha_t \geq 0$, $\beta_t \geq 0$). Thus, there exist some $\bar{\alpha}$ and $\bar{\beta}$ such that $\lim_{t\to\infty}\alpha_t = \bar{\alpha} \geq 0$ and $\lim_{t\to\infty}\beta_t = \bar{\beta} \geq 0$. As $t \to \infty$, we have $\bar{\alpha} + \bar{\beta} = |\bar{\alpha} - \bar{\beta}|$, which requires at least one of $\bar{\alpha}$ and $\bar{\beta}$ to be equal to zero. Suppose without loss of generality that $\bar{\alpha} = 0$. Since bargaining frontier has no horizontal or vertical segments, $\bar{\beta} = 0$ as well, which implies that our iteration algorithm converges to a single point (i.e., *IEC* is single valued). ■

Note that beyond showing that the solution is well-defined, this result provides a useful insight: the convergence of the iterative process can be interpreted as an explicit convergence of interests between E and EL. In addition to this, Proposition 2 will establish the fact that this convergence of interests satisfies an appealing property.