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Influence of boundary restraint on sound attenuation performance of a duct-membrane silencer

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ABSTRACT

Low frequency noise in duct is a challenge for the traditional passive noise control techniques. Recently, a so-called duct-membrane silencer has attracted much research attention due to its simple configuration and potential application, however, the current studies are merely limited to the cases in which just the classical boundary conditions are considered. Actually, as an important factor affecting the modal characteristics of the membrane, and the existing studies are not enough to fully understand the vibro-acoustic characteristics of such silencer with complicated boundary conditions. Motivated by this, in this paper, the structural–acoustic coupling model of duct-membrane system is established by a modified Fourier series method in combination with Rayleigh–Ritz procedure, in which the transverse elastic boundary restraints are taken into account. Energy principle is formulated for the vibro-acoustic coupling of such duct-membrane silencer to obtain the system matrix equation. Numerical results are then presented to validate the proposed model, and the influence of boundary restraining stiffness on sound attenuation performance is also studied. To the best of authors' knowledge, this work represents the first time that the elastic boundary restraints have been considered for such duct-membrane silencing system.

1. Introduction

The problem of low frequency noise control in duct, such as below 200 Hz, is often encountered in many engineering occasions, such as air conditioning and ventilation system [1]. Based on whether the secondary external energy is consumed or not, the noise control techniques can be categorized into two types: active and passive noise control. For the first one, it usually includes error sensor and secondary actuator, as well as the electronic controller [2]. Such system is effective for the low frequency noise control, however, it is believed that the introduction of these electronic devices will make the system complicated and expensive, at some circumstance, there will even be stability and reliability issues. Then, the realization of low frequency noise control through the passive means will be much more reliable and attractive.

For the traditional passive noise control, in essence, it attenuates the sound propagation in duct through dissipating the incoming sound into heat or the impedance mismatch. The first one is also called resistance silencer, where the sound absorbing material such as fiber is lined in the duct. The fiber damping effect will dissipate the sound energy into heat. But this kind of silencer just works well for the medium-to-high-frequency range. On the other hand, the impedance silencer is much more effective in low frequency, such as the expansion-chamber-type muffler. But, as summarized by Huang [3], it also has some problems, such as the existence of passbands, the bulkiness and the aerodynamic loss. So, low frequency duct noise is very difficult to deal with by traditional passive technique.

In recent years, Huang [3] proposed a novel duct-membrane silencer, which is based on the structural–acoustic coupling to achieve low frequency noise attenuation in duct. The tensioned membrane with clamped boundary condition is used as segment of the duct wall to reflect the grazing incident noise. Since there is no need of lining material and/or cross section variation like other traditional passive silencer does, such duct-membrane silencer is environmentally friendly, and has good flow-through characteristics. His investigation showed that the combination of noise reflection and damping of slow flexural waves can be very powerful in a broad frequency band. Subsequently, Huang theoretically and experimentally studied the membrane to be further backed by rigid-walled cavities, as should be the case in a practical silencer design, to form the so-called 'drum-like' or 'drum silencer' [4–6], which can achieve a satisfactory attenuation from low frequency to medium over an octave band. Huang and Choy [7] further extended the study into three-dimensional model in which the

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membrane has all four edges fixed. They found that the lateral tension was always detrimental to the silencing performance. Similarly, the silencer model with its segment of rigid duct replaced by a cavity-backed panel is also analyzed by Huang, in which both simply supported and clamped boundary conditions are investigated and compared [8,9]. The results show that, for a uniform plate, the optimal stop band is narrower than that of the simply supported configuration. Moreover, Choy and Huang [10] also explored the effect of flow on the drum-like silencer, their experimental results indicate that for the flow speed common in ventilation applications, there is no flow-induced flexural instability, and the possible noise radiated by the turbulence-induced vibration is insignificant.

In the aforementioned studies, most of them are confined to the classical boundary conditions for the flexible structure, such as simply supported and clamped. In fact, as one type of structural factors, boundary condition has an important effect on the modal characteristics of flexible structure as well as its vibro-acoustic coupling system. As found by Wang et al. [9], the boundary conditions also has significant influence on the silencing performance. From the practical point of view, the flexible structure will be not absolutely restrained in the ideal cases, while they are actually elastically restrained. On the other hand, a better understanding on the boundary restraint effect will be helpful for the optimal design of such silencer.

Motivated by the limitation in literature, a two-dimensional duct-membrane coupling model is established, in which the tensed membrane is elastically restrained in transverse direction at both ends. Different with the current studies that derive the solution from the differential governing equation, the energy principle is formulated for the whole vibro-acoustic coupling system [11,12], with the sound radiation process taken into account in terms of the work done for the vibrating membrane. An improved Fourier series method previously developed for the in-plane vibration analysis of rectangular plate [13] is employed for the construction of admissible function, with all unknown expansions coefficients determined via Rayleigh–Ritz procedure. Numerical results are then presented to validate the proposed model by comparing the calculated results with those from other approach in literature. The influence of boundary restraints on the silencing performance of duct-membrane system is studied and discussed based on the developed model.

2. Theoretical formulation

2.1. Model description

Let us consider a duct-membrane silencer, with its cross section along the longitudinal direction shown in Fig. 1, in which a segment of duct is covered by the tensing membrane with elastic boundary restraint. For the two-dimensional duct-membrane coupling system studied in this work, the string model will be employed accordingly. The coordinate system is also presented, in which the origin O is located on the left end of the membrane. L is the membrane length, and h is the duct height. In the duct upstream, there is an incident plane sound wave defined as $p_i = e^{-ik_0x+i\omega t}$, in which $\omega = 2\pi f$ is the angular frequency of the incident sound wave, and k_0 is equal to ω/c_0 with c_0 as the sound speed. For the 1-D membrane, elastic boundary springs are assumed to restrain the transverse vibration of string at both ends. k_1 and k_2 respectively denote the spring stiffness coefficients on the left and right end of the string. Any classical boundary condition can be easily realized by setting the stiffness coefficient into infinity or zero. For example, when both two spring stiffnesses are taken as infinity (a very large number in numerical calculation),

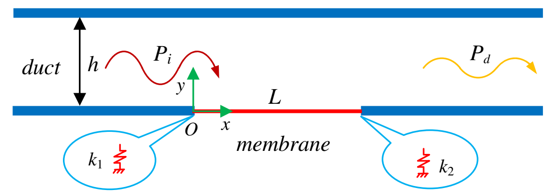

Fig. 1. A duct-membrane silencer with elastic boundary restraints.

the familiar fixed boundary condition will be obtained. When median value is set, it will be actually the elastic boundary restraint.

2.2. Energy formulation of the duct-membrane coupling system

For the vibro-acoustic coupling system illustrated in Fig. 1, mathematically, there will be two ways for the problem description. For the first one, the governing differential equations and boundary/coupling conditions are simultaneously used to formulate the whole coupled system. The other one is the integral form, namely energy formulation. When the admissible function is constructed smooth sufficiently, the later solution will be equivalent to the first one. In this work, the energy formulation is employed for the structural–acoustic description of duct-membrane system. The inherent benefit for such energy formulation is that the current model can be easily extended to include multiple vibro-acoustic subsystems.

The system Lagrangian for the current duct-membrane coupling model can be written as

$$L = U - T + W_P \tag{1}$$

in which, U and T are respectively the total potential energy and total kinetic energy associated with the transverse vibration of tensile string. W_P is the work done by sound pressure across the upper surface of string, and it represents the structural–acoustic interaction between the subsystems of duct and string.

For the total potential energy U, it actually includes two parts, namely, strain energy due to transverse deformation and the elastic potential energy stored in the elastic boundary restraints. It can be expressed as

$$U = \frac{1}{2}\int_0^L F\left(\frac{\partial u}{\partial x}\right)^2 dx + \frac{1}{2}[k_1u^2(0) + k_2u^2(L)] \tag{2}$$

here, F is the horizontal tensing force applied to the both ends of string.

The total kinetic energy associated with the string transverse vibration is

$$T = \frac{1}{2}\int_0^L \rho\left(\frac{\partial u(x,t)}{\partial t}\right)^2 dx \tag{3}$$

in which, ρ is the string mass density of unit length.

During the calculation of work done by the sound pressure W_P, the pressure across the upper surface of the string is the superposition of two parts, namely

$$P = P_i + P_{rad} \tag{4}$$

here, P_i is the sound pressure propagating from the duct upstream, and P_{rad} is the sound radiation into the duct due to the string flexural vibration.

The radiated sound pressure from the vibrating structure in duct can be predicted by using the following formula [14]:

$$P_{rad}(x,y) = \frac{\rho_0}{2h}\sum_{m=0}^{\infty} c_m\psi_m(y)\int_0^L \psi_m(y')v(x',y') \times \left[H(x-x')e^{-ik_m(x-x')} + H(x'-x)e^{+ik_m(x-x')}\right]dx' \tag{5}$$