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A structured method for the design-for-frequency of a brace-soundboard system using a scalloped brace

Patrick Dumond *, Natalie Baddour

Department of Mechanical Engineering, University of Ottawa, 161 Louis Pasteur, CBY A205, K1N 6N5 Ottawa, Canada

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ABSTRACT

Design-for-frequency of mechanical systems has long been a practice of iterative procedures in order to construct systems having desired natural frequencies. Especially problematic is achieving acoustic consistency in systems using natural materials such as wood. Inverse eigenvalue problem theory seeks to rectify these shortfalls by creating system matrices of the mechanical systems directly from the desired natural frequencies. In this paper, the Cayley–Hamilton and determinant methods for solving inverse eigenvalue problems are applied to the problem of the scalloped braced plate. Both methods are shown to be effective tools in calculating the dimensions of the brace necessary for achieving a desired fundamental natural frequency and one of its higher partials. These methods use the physical parameters and mechanical properties of the material in order to frame the discrete problem in contrast to standard approaches that specify the structure of the matrix itself. They also demonstrate the ability to find multiple solutions to the same problem. The determinant method is found to be computationally more efficient for partially described inverse problems due to the reduced number of equations and parameters that need to be solved. The two methods show great promise for techniques which could lead to the design of complex mechanical systems, including musical instrument soundboards, directly from knowledge of the desired natural frequencies.

1. Introduction

Designing mechanical systems to achieve desired frequencies has long been a trial-and-error exercise. The approach generally involves forward model design and iteration of design parameters until desired frequencies are achieved. This approach, while effective, is inefficient. A better approach would be to design the system directly from the desired frequencies, thus a structured method is desirable. Furthermore, iterative forward model design only works well for typical engineered materials such as metals and plastic, which demonstrate dependable mechanical properties. For systems using materials with highly variable mechanical properties, such as wood, this iterative design approach proves to be unpractical. In essence, in order to design-for-frequency using material specimens that demonstrate large variations in mechanical properties, it is much easier to design the system directly from those desired frequencies.

A small field of study, known as inverse eigenvalue problems, attempts to address such problems. A discrete inverse eigenvalue problem attempts to construct matrices, representative of physical systems, directly from a set of given eigenvalues (natural frequencies) [1]. Discrete inverse eigenvalue theory uses knowledge of matrix algebra and numerical methods in order to create matrices which yield the desired frequency spectrum (sets of eigenvalues). Thus, using these methods, it should be possible to design a system (represented by a set of mass and stiffness matrices) from a set of desired frequencies.

It is well known that inverse eigenvalue problems are ill-posed, meaning that many matrices exist which satisfy a single set of eigenvalues [2]. In engineering, the existence of multiple solutions could potentially be beneficial, giving the designer many design options. However, it is important to keep in mind that in order to ensure that a design is physically realizable, physical constraints must be included. Most methods developed for inverse eigenvalue problems stem from the field of structured matrix theory (e.g. Jacobi, band matrices and other matrix forms) using proven numerical algorithms to reconstruct unknown matrices from a full or partial set of desired eigenvalues [3–10]. Thus the typical approach is to limit the number of solutions by framing the inverse problem within a pre-determined matrix structure. Although structured matrices generally imply various physical constraints,

* Corresponding author.
E-mail addresses: pdumo057@uottawa.ca (P. Dumond), nbaddour@uottawa.ca (N. Baddour).

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very few methods exist for matrices which have a more general form. One of the goals of this paper is to demonstrate the use of the recently proposed inverse eigenvalue technique using the Cayley–Hamilton theorem [11]. This approach is particularly interesting because it enables the solution of *any* matrix structure. Thus, the solution of any matrix can be obtained from a set of eigenvalues and applying the physical constraints to the matrix structure becomes an exercise in the forward modeling process. The system can then be limited by the material of choice rather than a certain matrix structure.

From the Cayley–Hamilton theorem method, we derive a second method, which we refer to as the determinant method and which will be explained herein. In certain situations, including the problem presented in this paper, only a select few frequencies, as opposed to the full spectrum, are required to be specified. These problems are known as partially described problems. The determinant method approach has the benefit of solving partially described systems using fewer equations.

In this paper, we apply the Cayley–Hamilton theorem method, as well as the determinant method to the problem of designing a scalloped shaped brace on a simple rectangular plate in order to achieve desired system frequencies. This brace-plate model is chosen to demonstrate the validity of the methods because it has been previously analyzed to examine the effects of the scalloped brace on the natural frequencies of a brace-soundboard system [12] and thus the forward problem is well understood. In the prior paper, the dimensions of the brace were adjusted by trial and error. In this paper, structured methods will be used to calculate the requisite dimensions of the scalloped brace required in order to achieve the desired system frequencies. Results are then validated by comparison to those previously presented.

2. Model

2.1. The mechanical system

The model used during the analysis is based on a typical section of a guitar soundboard structurally reinforced by a single brace along the weaker grain direction and first developed in [12] to explore the effects that a scalloped shaped brace has on the frequencies of a system. The layout of the model is shown in Fig. 1.

Since this prior paper demonstrated that scalloped-shaped braces typically used by musical instrument makers can be used to control two natural frequencies of the combined brace-plate system simultaneously, the same brace shape will be used herein. A scalloped shaped brace can be seen in Fig. 2.

2.2. Problem statement

The problem we seek to solve can be stated as follows: calculate the dimensions of the scalloped brace so that the brace-plate

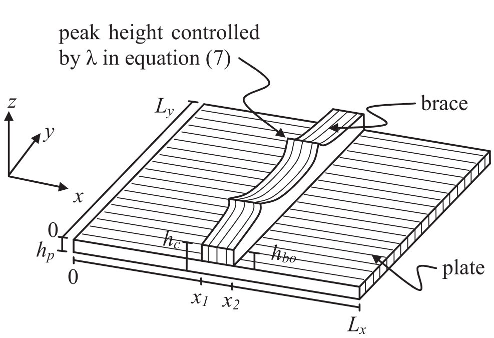

Fig. 1. Orthotropic plate reinforced with a scalloped brace.

Fig. 2. Shape of a scalloped brace.

system, described by a mass matrix M and a stiffness matrix K, has a specified given fundamental frequency and a specified higher partial (two frequencies specified) as the radial stiffness of the plate (E_R) varies. The variation of the radial stiffness of the plate is the means by which we model the natural occurring specimen-to-specimen variations in the material properties of wood. Although the methods presented herein work when any number of mechanical properties are varied. Thus, for a plate specimen with a given radial stiffness, we demonstrate how to calculate the corresponding requisite brace dimensions that will ensure two frequencies in the plate's spectrum are at specified values.

Since the model presented in [12] is made of wood, all mechanical properties of the system are assumed to be a function of the material radial stiffness E_R, which is assumed known and given but which tends to vary from specimen to specimen. This inter-specimen variation of the radial stiffness is the cause of acoustic inconsistencies in the manufacture of musical instruments in spite of tight dimensional manufacturing tolerances [13]. All dimensional (geometric) properties of the brace-plate system are assumed to be specified and fixed except for the base thickness of the brace h_{bo} and the scallop peak height adjustment factor λ, the design variables to be calculated.

2.3. Forward model

Rather than choosing a pre-determined matrix structure, the shape of the matrix is defined through the process of constructing a forward system model, taking into account the fixed geometric and mechanical properties of the physical system, but leaving as variables to be solved the parameters that remain under the control of the designer.

Although any discretization method could be used, following on our prior paper, the assumed shape method is used to discretize the forward model. The assumed shape method is intended as a theoretical model for generating the M and K matrices in order to demonstrate the inverse methods presented in Sections 2.4.1 and 2.4.2. The assumed shape method is an energy method which applies global elements to the kinetic and strain energy equations in order to determine the mass and stiffness matrices representative of the system [14]. The kinetic energy of a conservative, simply supported orthotropic Kirchhoff plate is used. Although this is an accurate assumption for the plate, it may imply a certain error at the location of the brace where thicker plate theories may be more appropriate. The kinetic energy is separated into three sections in order to take into account the presence of the brace, as shown in Fig. 1, and is given by

$$T = \frac{1}{2}\int_0^{x_1}\int_0^{L_y} \dot{w}^2 \rho_p \,dy dx + \frac{1}{2}\int_{x_1}^{x_2}\int_0^{L_y} \dot{w}^2 \rho_c \,dy dx + \frac{1}{2}\int_{x_2}^{L_x}\int_0^{L_y} \dot{w}^2 \rho_p \,dy dx \quad (1)$$

where L_x and L_y are the dimensions of the plate in the x and y directions respectively. The dot above the transverse displacement variable w represents the time derivative, ρ is the mass per unit area of the plate such that

$$\rho_p = \mu \cdot h_p \text{ and } \rho_c = \mu \cdot h_c, \quad (2)$$

μ is the material density and h_p and h_c are the thickness of the plate and combined brace-plate sections, respectively.