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Numerical characterization of acoustic scattering coefficients of one-dimensional periodic surfaces

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ABSTRACT

In room acoustics design, one-dimensional periodic surfaces, such as ribbed or corrugated walls, are ordinary used to scatter reflected sound and to increase sound field diffuseness. There exists some measured data of random-incidence scattering coefficient for periodic surfaces, however a substantial database is not yet provided. Instead of the measurement, numerical analysis in a free field is useful to calculate directional scattering coefficients. This paper numerically investigates the characteristics of random- and normal-incidence scattering coefficients of one-dimensional periodic surfaces, focusing on the effects of shape, height and width of surface profile. Additionally, 1/4-scale model measurements are conducted for typical profiles in order to verify the numerical results. Finally, optimal height-to-period ratios, maximizing random- or normal-incidence scattering coefficient, are found for sinusoidal, triangular and rectangular profiles, respectively.

1. Introduction

Acoustic scattering from wall surfaces is an important factor related to the acoustic quality of rooms, where its applications show the effects of preventing echoes, promoting spaciousness, reducing coloration and controlling modes in many architectural spaces [1]. Until now, several kinds of indices to evaluate reflection characteristics of surfaces are proposed [2–6], and as one of the indices, the scattering coefficient (see. Fig. 1) is defined as the ratio of the non-specularly reflected acoustic energy to the totally reflected energy [6,7]. The coefficient is very useful to enhance the accuracy of geometric room acoustics simulation, thus it is already included in some commercial software programs [8,9].

For quantifying the scattering coefficients, the reverberation room method for measuring random-incidence values, originally developed by Vorländer and Mommertz [6], has been standardized by ISO 17497-1 [10]. Although its measurement setups on test sample shape, mounting, rotating and time variance are recently almost robust [11–16], measurement methods for incidence-angle-dependent values are not yet fully established. Recently, the authors have proposed a new laboratory method for measuring normal-incidence scattering coefficients [17], which would be useful to evaluate suppression of flutter echoes between parallel walls, but still under development.

As an alternative to the above measurements, Kosaka and Sakuma [18,19] have realized numerical evaluation of directional scattering coefficients in a free field, based on the Mommertz's definition [7], and employing the boundary element method (BEM). Up to now, the numerical evaluation is almost feasible in the full frequency range, thus it enables various parametric studies for surface design.

There are several experimental researches regarding the characterization of scattering coefficients for diffuser design. Jeon et al. [20] measured the coefficients of hemispherical and cubic blocks, with different heights, coverage densities and arrangements. Kim et al. [21] investigated the effects of height and coverage density of one- and two-directional grooves, and also discussed the relationship between scattering and absorption coefficients. Tsuchiya et al. [22] conducted scale model measurements to examine the effects of surface profile and absorbent finish for rib structures, and that of arrangement for block structures. Besides, in the early studies [11–14,23], the coefficients of sine-shaped surfaces were often measured as benchmark samples. However, a substantial database of scattering coefficient is not yet provided for practical use in room acoustics design. Furthermore, any guideline of surface design is required, particularly for one-dimensional periodic surfaces, such as ribbed or corrugated walls in ordinary use.

In this paper, characteristics of random- and normal-incidence scattering coefficients of one-dimensional periodical surfaces are numerically investigated, focusing on the effects of shape, height and width of surface profile. Firstly, as popular surface profiles,

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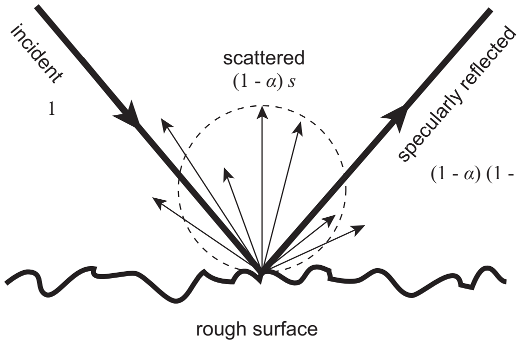

Fig. 1. Scattering from rough surfaces.

the effects of three types of shapes with sinusoid, triangles and rectangles are examined. Secondly, the effects of structural height are tested for the three types, with changing the ratio of height to period from 10% to 50%. Thirdly, for the type with rectangles, the effect of width is tested with changing the ratio of width to period from 0% to 75%. Additionally, 1/4-scale model measurements are conducted for typical profiles in order to verify the numerical results. Furthermore, optimal height-to-period ratios that maximize random- and normal-incidence scattering coefficients are found for sinusoidal, triangular and rectangular profiles, respectively.

2. Numerical determination of scattering coefficients

2.1. Calculation of reflection directivity with BEM

Mommertz presented a method for evaluating directional scattering coefficients in a free field, where the coefficient is given by the correlation between two reflection directivities from a test sample and a reference flat plate of an equal area [7]. To simulate the free field method, the application of the BEM is quite effective, especially the so-called indirect BEM in the normal derivative form (hypersingular form), assuming that the sample and the reference plate are perfectly rigid and have negligible thickness.

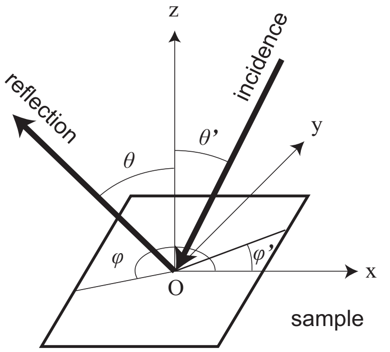

Fig. 2. Geometry of the numerical model.

In the numerical model, a plane wave impinges on a surface of the sample or the reference plate in the free field as shown in Fig. 2. With sampling N directions in the upper hemisphere at an equal solid angle, applying the BEM in the normal derivative form on every incidence condition gives the following matrix equation [18]:

$$\boldsymbol{A}\cdot[\tilde{\boldsymbol{p}}_1,\tilde{\boldsymbol{p}}_2,\cdots,\tilde{\boldsymbol{p}}_N]=[\boldsymbol{d}_1,\boldsymbol{d}_2,\cdots,\boldsymbol{d}_N], \tag{1}$$

with

$$A_{ij}=\iint_{e_j}\frac{\partial^2 G(\boldsymbol{r}_i,\boldsymbol{r}_q)}{\partial\boldsymbol{n}_i\partial\boldsymbol{n}_q}dS_q,\ \text{and}\ d_{il}=\frac{\partial}{\partial\boldsymbol{n}_i}\exp(-j\boldsymbol{k}_l\cdot\boldsymbol{r}_i),$$

where $\tilde{\boldsymbol{p}}_{jl}$ is the sound pressure difference between the two sides of the j-th element e_j, and $\boldsymbol{k}_l$ is the wave number vector for the l-th incidence condition with the incidence angles $(\theta'_l,\ \varphi'_l)$, G is the Green's function, $\boldsymbol{r}_i$ and $\partial/\partial\boldsymbol{n}_i$ denote the location vector and the normal derivative at the i-th node, respectively. Thus, by solving the above equation, the sound pressure differences for every incidence condition are given.

In the post process, considering that the receiving point for every direction is sufficiently far from the center of the sample at an equal distance, the reflected sound pressure distribution for every incidence condition is calculated by the following equation:

$$[\boldsymbol{p}_1,\ \boldsymbol{p}_2,\ \cdots,\ \boldsymbol{p}_N]=-\boldsymbol{H}\cdot[\tilde{\boldsymbol{p}}_1,\ \tilde{\boldsymbol{p}}_2,\ \cdots,\ \tilde{\boldsymbol{p}}_N], \tag{2}$$

with

$$H_{mj}=\iint_{e_j}\frac{\partial G(\boldsymbol{r}_m,\boldsymbol{r}_q)}{\partial\boldsymbol{n}_q}dS_q,$$

where $\boldsymbol{r}_m$ is the location vector of the m-th receiving point with the reflection angles $(\theta_m,\ \varphi_m)$, and p_{ml} is the reflected sound pressure at the m-th receiving point for the l-th incidence condition.

2.2. Calculation of scattering coefficients

After the two reflection directivities of the sample and the reference plate are calculated by the BEM, according to Mommertz's definition [7], the directional scattering coefficient for each incidence condition is given by

$$S(\theta'_l,\ \varphi'_l)=1-\frac{\left|\sum_{m=1}^{N}p_{ml}\cdot\hat{p}^*_{ml}\right|^2}{\sum_{m=1}^{N}|p_{ml}|^2\cdot\sum_{m=1}^{N}|\hat{p}_{ml}|^2}, \tag{3}$$

where p_{ml} and $\hat{p}_{ml}$ are the complex sound pressure reflected by the sample and the reference plate. Subsequently, according to Paris' formula, the random-incidence scattering coefficient can be obtained by statistical averaging of directional coefficients.

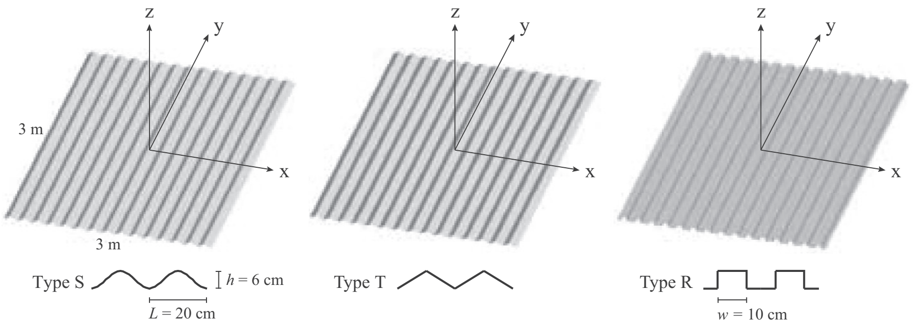

Fig. 3. Three types of one-dimensional periodic surfaces (15 periods). Type S: sinusoids; Type T: triangles; and Type R: rectangles.