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A global error estimator for the uncertainty of a multi-channel spectral analysis

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ABSTRACT

A global estimator of the uncertainty of the average frequency response function in multi-channel spectral analysis measurements is proposed. The proposed global estimator is a generalization of the random error estimator of the frequency response function magnitude of a single-input–single-output system. In principle, the signal-to-noise ratio (and thus the quality of the frequency response function estimation) is increasing with increasing number of averages M, according to $\sqrt{M}$. However, in the situation that, for practical reasons, there is a maximum imposed upon the total measurement time T_{max}, it is clear that there is a trade-off between the number of averages M and the record length T (s) that is used to obtain an estimate of a single-average-frequency-response-function. There is a choice between a few long records or many short records, with the requirement that, assuming zero overlap, the number of averages M times the record length T may not exceed the total available measurement time, i.e. $M \times T \leqslant T_{max}$. In addition to the existence of such an optimum, a minimum record length is required as well which is related to the reverberation time of the system. The newly proposed global estimator is used to determine the optimal record length of a multi-channel system, such that a minimum error of the average frequency response function is obtained. It is also shown by experimental results that indeed the minimum allowable record length is related to the reverberation time of the system being measured.

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1. Introduction

In this article a global estimator is proposed. For demonstration purposes the estimator is applied to the multi-channel spectral analysis of a vibrating building element. The measurements were carried out by means of a scanning laser Doppler vibrometer and were intended to determine the radiated sound power at low frequencies (see also [1]). An interesting complicating factor in this experiment is that the building element is excited by a reverberant acoustic field, which exhibits a random characteristic in time. By looking for a minimum of the proposed global estimator, an optimum record length is sought, for a given reverberation time of the room. This is a classical problem, discussed by Piersol in 1978 [2], and later also by Jacobsen and Nielsen in 1987 [3], where it was concluded that the analysis record lengths should be at least as long as the reverberation time to avoid serious errors in the coherence estimate. The measurements also indeed demonstrate nicely that the minimum record length is required to be longer than the reverberation time of the system.

2. Theory

A multiple-channel-output system is considered as illustrated in Fig. 1. A building element is excited by an acoustic field, as shown, resulting in a response being measured at different positions, $i = 1 \dots N$. An additional position r is being probed simultaneously with each of the other positions, thus serving as a reference, allowing the responses at positions $i = 1 \dots N$ to be measured sequentially in time, retaining the phase information of the response at the individual positions (assuming the building element to respond stationary and time-independent). As we assume the exciting acoustic field to be caused by a single sound source (loudspeaker), we are dealing with a single-input-multiple-output system, for which a single reference point is sufficient. In case of a multiple-input-multiple-output system more reference points will be necessary.

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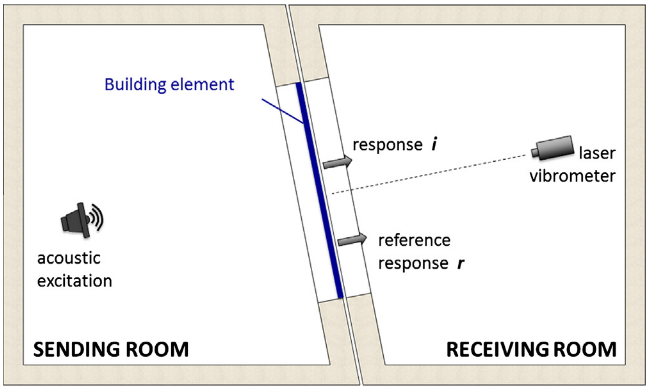

Fig. 1. Test facility measurement set-up.

In the following the auto-spectra of the response of the structure at different positions is denoted by S_{ii}, with $i = 1 \ldots N$, the auto-spectrum of the reference signal is denoted by S_{rr}, and the cross-spectra between the response of the structure at different positions and the reference signal are denoted by S_{ir}, with $i = 1 \ldots N$. The frequency response function between the response of the structure at different positions and the one of some reference signal can then be estimated using the H^1-optimal estimate or the H^2-optimal estimate, respectively [4]:

$$H^1_{ir} = \frac{S_{ri}}{S_{rr}} \tag{1}$$

$$H^2_{ir} = \frac{S_{ii}}{S_{ir}} \tag{2}$$

It is well known that measurement noise is better suppressed when averaging cross-spectra, compared to averaging auto-spectra [4–6]. Taking a cross-spectrum enhances common frequency components in both signals that have a consistent phase relationship. Uncorrelated signal components average to zero. Since auto-spectra are positive valued, summing them accumulates not only correlated signal components, but also uncorrelated ones.

In view of this, as the estimate of frequency response functions is based upon both cross-spectra and auto-spectra, it is best to choose the H-estimate with the least noisy autocorrelation factor, i.e. use H^1 in case less noise is expected on the reference auto-spectrum S_{rr} and use H^2 if less noise is expected on the response auto-spectra S_{ii}.

2.1. Global coherence and global ϵ

A quantity that plays an important role in the statistical uncertainty of the averaged frequency response function of a single-input–single-output system is the coherence of the frequency response function, which is defined as [1]

$$\gamma_{ri} = \frac{|S_{ri}|^2}{S_{rr}S_{ii}} \tag{3}$$

Given the coherences γ_{ri} related to each frequency response function H_{ri}, the normalized standard deviations due to random errors in single-input–single-output systems are given by [1]:

$$\epsilon_{H_{ri}} = \frac{1}{\sqrt{2M}}\sqrt{\frac{1-\gamma_{ri}}{\gamma_{ri}}} \tag{4}$$

where M is the number of averages.

As mentioned above, the goal of this paper is to introduce a global coherence function that considers the quality of all measurements simultaneously, as well as a global statistical uncertainty on the average frequency response function. As the coherence function γ_{ri} as defined above can be seen as the ratio between the energy in the auto-spectrum of signal i that is coherent with the reference signal, $\gamma_{ri}S_{ii}$, and the total energy in the auto-spectrum of signal i, S_{ii}, we propose the following expression for the global coherence:

$$\gamma^{global} = \frac{\langle \gamma_{ri} S_{ii}\rangle_N}{S_{iiN}} \tag{5}$$

where $\langle \ldots \rangle_N$ denotes the average over all N measurement points. The thus defined global coherence can be viewed as the ratio between the mean coherent auto-spectrum by the mean raw auto-spectrum.

In addition, we propose an estimate for the statistical uncertainty of the average frequency response function as follows:

$$\epsilon^{global}_{H_{ri}} = \sqrt{\frac{\langle \epsilon^2_{H_{ri}} |H^2_{ri}|\rangle_N}{N\langle |H_{ri}|\rangle_N^2}} = \frac{\sqrt{\frac{\langle \epsilon^2_{H_{ri}} |H_{ri}|^2\rangle_N}{N}}}{\langle |H_{ri}|\rangle_N} \tag{6}$$

Indeed, the variance of the modulus of the average frequency response function is given by

$$\mathbb{V}\{\langle |H_{ri}|\rangle_N\} = \frac{1}{N}\langle \mathbb{V}\{|H_{ri}|\}\rangle_N \tag{7}$$

while

$$\mathbb{V}\{|H_{ri}|\} = |H_{ri}|^2 \epsilon^2_{H_{ri}} \tag{8}$$

Hence, assuming a gaussian distribution, it can be stated with 95% certainty that the actual value of $\langle |H_{ri}|\rangle_N$ lies between $\langle |H_{ri}|\rangle_N - 2\epsilon^{global}_{H_{ri}}\langle |H_{ri}|\rangle_N$ and $\langle |H_{ri}|\rangle_N + 2\epsilon^{global}_{H_{ri}}\langle |H_{ri}|\rangle_N$.

Note that in [1] (Section 8.2) a multiple coherence function was defined for a system with multiple inputs and a single output, which provides a measure of the linear dependence between a collection of inputs and an output, independent of the correlation among the inputs. However, to the best of our knowledge, a global coherence as defined above for a single input multiple output system, has not been proposed before. The same is true for the global error of the average of the frequency response function.

2.2. The multi-pass estimator

In the situation that the system being investigated responds stationary and time-independent, the frequency response functions $H_{ri},\ i = 1, \ldots, N$, can be measured sequentially in time using a multi-pass measurement scheme, which puts less heavy demands on the measurement and data acquisition system in terms of the required number of channels. Assuming that a H^1-type of estimate of the frequency response functions is required, a fixed reference signal, here denoted as the signal measured at point r, should be used in order to retain the phase information at each point $i = 1 \ldots N$, using Eq. (2) for the estimation of the frequency response function:

$$H^1_{ri} = \frac{S_{ri}}{S_{rr}} \tag{9}$$

By estimating the frequency response function between this reference signal and the response of the structure at different positions, being measured sequentially in time, the phase between the response at the different measurement points and the reference can be determined, and thus also the phase between the individual responses.

A particularity of such a multi-pass measurement scheme is that the reference signal is acquired at each measurement pass, whilst all other signals are recorded during one pass only. Having N measurement passes to determine the N components of the

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