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Valuing equity-indexed annuities with icicled barrier options

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A B S T R A C T

Inspired by the recent popularity of autocallable structured products, this paper intends to enhance equity-indexed annuities (EIAs) by introducing a new class of barrier options, termed *icicled barrier options*. The new class of options has a vertical (icicled) barrier along with the horizontal one of the ordinary barrier options, which may act as an additional knock-in or knock-out trigger. To improve the crediting method of EIAs, we propose a new EIA design, termed *autocallable EIA*, with payoff structure similar to the autocallable products except for the minimum guarantee, and further investigate the possibility of embedding various icicled barrier options into the plain point-to-point or the ratchet EIAs. Explicit pricing formulas for the proposed EIAs and the icicled barrier options are obtained under the Black–Scholes model. To the purpose, we derive the joint distribution of the logarithmic returns at the icicled time and the maturity, and their running maximum. As an application of the well-known reflection principle, the derivation itself is an interesting probability problem and the joint distribution plays a key role in the subsequent pricing stage. Our option pricing result can be easily transferred to EIAs or other equity-linked products. The pricing formulas for the EIAs and the options are illustrated through numerical examples.

1. Introduction

Equity-indexed annuities (EIAs) have gained a great deal of popularity since they have been first offered by Keyport Life in 1995. According to the recent LIMRA market survey, the annual EIA sales or the annual fixed income annuity sales are still growing. However, the prolonged period of low interest rate creates a conflicting environment which makes the EIA participation rate lower but the equity market participation more attractive. Hence, in order to improve the crediting method and add more flavors, we introduce a new type of EIA designs and barrier options inspired by autocallable structured products or autocallable equity-linked security (ELS). Roughly speaking, the autocallable ELS invests in the equity market and redeems the original investment with a pre-specified rate of return at pre-specified autocall dates earlier than the maturity if the value of the underlying asset satisfies a certain criterion. Early redemption with a higher rate of return has made the autocallable ELS appealing to the investors. Therefore, as explained later, we borrow the concept of autocall date into our new EIA designs as a way of locking in a higher rate of return before maturity or increasing the participation rates.

Our EIA designs are closely related to a new class of barrier options with additional vertical barrier superimposed upon the horizontal one of the ordinary barrier options. In Fig. 1, we depict a sample path of the underlying asset and two barriers of this new option. The initial value of the underlying asset is set at 100. The option has a horizontal (up) barrier of 110 for its entire lifetime and an additional vertical barrier of 105 in the middle. Since the barriers resemble an icicle, we coin the name

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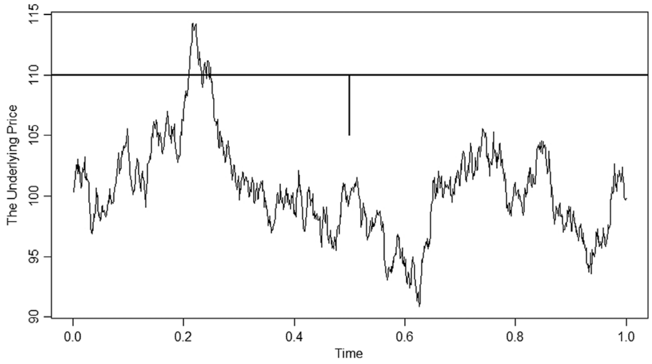

Fig. 1. A sample path of the underlying asset price with its initial value set at 100, and an icicled barrier option with a horizontal barrier of 110 and an icicled barrier of 105 at $t = 0.5$.

icicled barrier option. As with the ordinary barrier option, the icicled barrier can serve as a knock-in or knock-out trigger if it can be reached by the underlying asset. For instance, let us consider an up-and-in icicled barrier option with the same barrier as in Fig. 1. If the price of the underlying asset follows the sample path in Fig. 1, the icicled barrier option would come into existence and give the pre-specified payoff to the option holder.

Our primary interest of this paper lies in the derivation of explicit pricing formulas of the icicled barrier options under the celebrated Black–Scholes model. To the purpose, it is essential to have the joint distribution of the logarithmic returns at the icicled time and the maturity, and their running maximum. Obviously, whether an icicled barrier option comes into existence or not depends upon the first and the last variables of the distribution, but the option payoff could be related to the second one. As with the ordinary barrier option, this paper will consider eight types of icicled barrier options according to the combinations of (up, down), (in, out), and (call, put), with the payoff being determined by the underlying asset value at maturity.

Although we consider only one spike of icicle for simplicity, it is possible to equip the icicled barrier options with more spikes and generalize our mathematical development. Moreover, we may redesign the option payoff without much difficulty once we have the joint distributions of the logarithmic returns at multiple icicled spikes and maturity, and their running maximum. Therefore, our icicled barrier options bestow a considerable flexibility on the payoff structure while allowing for explicit pricing formulas. At first glance, the icicled barrier option might look cheaper than the corresponding ordinary barrier option. However, this is not always the case if it is a knock-in barrier option. Therefore, as in the autocallable ELS with down-and-in puts short, various equity-linked securities or annuities could be made more attractive by shorting various icicled barrier options.

In this paper, we derive the joint distributions in two different ways based on the well-known reflection principle. As can be seen later, the derivation itself is an interesting probability problem. In the first proof, we follow the steps used in the discussion paper of Huang and Shiu (2001), where the joint distribution of the logarithmic returns at maturity and their running maximum is obtained by the method of Esscher transform. Extending their ideas, we obtain the joint distributions of our target. In the second, we exploit the law of iterated expectations and confirm the first derivation. Once the joint distributions are ready, it would be straightforward to derive the relevant option pricing formulas through the expectation calculation. In the meantime, the Esscher transform will demonstrate its time-honored usefulness for such expectation calculation. If we take it into account that the icicled barrier option has a vertical barrier, it seems somewhat surprising that they can be priced in terms of closed form expressions.

Our option pricing method is completely probabilistic, so the background knowledge regarding the partial differential equation (PDE) is not necessary. Unlike the previous literature where the autocallable ELS are evaluated by the PDE approach or the Monte Carlo simulation, we exploit the properties of Brownian motion and the method of Esscher transform. For some previous works about the autocallables, we refer the readers to Alm, Harrach, Harrach, and Keller (2013), Bouzoubaa and Osseiran (2010), Deng, Mallett, and McCann (2011), and the papers cited therein. The Esscher transform was originally developed to approximate the aggregate claim amount distribution, but has been extensively used to evaluate various options embedded in equity linked insurance products since the work of Gerber and Shiu (1994). For instance, see Gerber, Shiu, and Yang (2012), Lee (2003), Ng and Li (2011), and Tiong (2000) to name a few.

The remainder of this paper is organized as follows. In Section 2, we formulate our mathematical framework and review some relevant materials including the Esscher transform. In Section 3, we derive the joint distribution of our interest and other joint probabilities in terms of the bivariate standard normal distributions. In Section 4, we find explicit pricing formulas for various icicled barrier options. In Section 5, we introduce a new EIA design with its payoff similar to the autocallable products and derive the explicit pricing formula. In Section 6, we explore the possibility of embedding icicled barrier options into the plain point-to-point and the ratchet EIAs. At the end of each section, we further investigate our pricing formulas through numerical examples. Section 7 concludes this paper with some comments about the future research. Finally, in Appendix A, we provide an alternative derivation for the joint distribution.