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Modeling spatial anisotropy via regression with partial differential regularization

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Abstract

We consider the problem of analyzing spatially distributed data characterized by spatial anisotropy. Following a functional data analysis approach, we propose a method based on regression with partial differential regularization, where the differential operator in the regularizing term is anisotropic and is derived from data. We show that the method correctly identifies the direction and intensity of anisotropy and returns an accurate estimate of the spatial field. The method compares favorably to both isotropic and anisotropic kriging, as tested in simulation studies under various scenarios. The method is then applied to the analysis of Switzerland rainfall data.

Keywords: Finite elements, Functional data analysis, Parameter cascading, Penalized regression

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1. Introduction

Many, if not most, spatial phenomena are characterized by spatial anisotropy. In biology, anisotropy is naturally induced by the arrangement and orientation of fibers and cells in a tissue, or by the morphology of the organs; in meteorology, it may be caused by the presence of winds and sea streams, or by the orography of the region under study; in geology, by the process of sedimentation. Figure 1 depicts a dataset of 467 daily rainfall measurements recorded in Switzerland on May 8, 1986; this dataset was used for the Spatial Interpolation Comparison 97 [9]. The size and color of point markers represent the value of the rainfall at each location, highlighting a strong spatial anisotropy, with higher rainfall values alternating with lower rainfall values along elongated regions oriented in the northeast-southwest direction.

In this work we adopt a functional data analysis approach and propose to model the spatial anisotropy via regression with partial differential regularization. Ramsay [21], Sangalli et al. [24] and Wood et al. [28] consider spatial regression with a roughness penalty that involves the Laplacian of the spatial field: this partial differential operator provides a simple and isotropic measure of the curvature of the spatial field; its use in the regularizing term induces an isotropic smoothing effect. Thin plate splines [26] and bivariate splines over triangulations [14] offer other classical and recent proposals of spatial isotropic smoothing defined as regression with differential regularization. Azzimonti et al. [1] extend the method in Ramsay [21] and Sangalli et al. [24] to the case where the regularizing term involves a more general partial differential equation (PDE) that induces an anisotropic and non-stationary smoothing. In particular, Azzimonti et al. [1] assume that the PDE in the regularizing term is suggested by prior knowledge of the phenomenon under study, coming for instance from the physics or morphology of the problem; the parameters of the PDE are consequently fixed considering their physical meaning.

We here instead assume no prior knowledge of the spatial variation of the considered problem; moreover, we do not assume the existence of a physical law governing the system. We though use the PDE in the regularizing term to model the spatial variation of the phenomenon, learning the anisotropy directly from the data. Specifically, we consider PDEs that induce a stationary anisotropic smoothing effect; the parameters in the PDE determine the

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