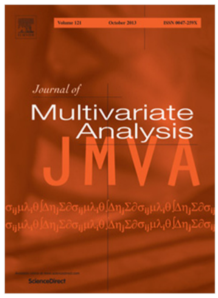
Accepted Manuscript



Risk contagion under regular variation and asymptotic tail independence

Bikramjit Das, Vicky Fasen-Hartmann

PII: S0047-259X(17)30767-4
DOI: https://doi.org/10.1016/j.jmva.2017.12.004
Reference: YJMVA 4313

To appear in: *Journal of Multivariate Analysis*

Received date : 30 March 2016

Risk contagion under regular variation and asymptotic tail independence

Bikramjit Das[a], Vicky Fasen-Hartmann[b]

[a] *Singapore University of Technology and Design, 8 Somapah Road, Singapore 487372*
[b] *Karlsruher Institut für Technologie, Englerstraße 2, 76131 Karlsruhe, Germany*

Abstract

Risk contagion concerns any entity dealing with large scale risks. Suppose $\boldsymbol{Z} = (Z_1, Z_2)$ denotes a risk vector pertaining to two components in some system. A relevant measurement of risk contagion would be to quantify the amount of influence of high values of Z_2 on Z_1. This can be measured in a variety of ways. In this paper, we study two such measures: the quantity $\mathrm{E}\{(Z_1 - t)_+ | Z_2 > t\}$ called Marginal Mean Excess (MME) as well as the related quantity $\mathrm{E}(Z_1 | Z_2 > t)$ called Marginal Expected Shortfall (MES). Both quantities are indicators of risk contagion and useful in various applications ranging from finance, insurance and systemic risk to environmental and climate risk. We work under the assumptions of multivariate regular variation, hidden regular variation and asymptotic tail independence for the risk vector $\boldsymbol{Z}$. Many broad and useful model classes satisfy these assumptions. We present several examples and derive the asymptotic behavior of both MME and MES as the threshold $t \to \infty$. We observe that although we assume asymptotic tail independence in the models, MME and MES converge to infinity under very general conditions; this reflects that the underlying weak dependence in the model still remains significant. Besides the consistency of the empirical estimators we introduce an extrapolation method based on extreme-value theory to estimate both MME and MES for high thresholds t where little data are available. We show that these estimators are consistent and illustrate our methodology in both simulated and real data sets.

Keywords: Asymptotic tail independence, consistency, expected shortfall, heavy-tail, hidden regular variation, mean excess, multivariate regular variation, systemic risk.

1. Introduction

The presence of heavy-tail phenomena in data arising from a broad range of applications spanning hydrology [2], finance [37], insurance [16], internet traffic [8, 34], social networks and random graphs [5, 14] and risk management [9, 24] is well documented. Since heavy-tailed distributions often entail non-existence of some higher order moments, measuring and assessing dependence in jointly heavy-tailed random variables poses a few challenges. Furthermore, one often encounters the phenomenon of asymptotic tail independence in the upper tails, which means that given two jointly distributed heavy-tailed random variables, joint occurrence of very high (positive) values is extremely unlikely.

In this paper, we look at heavy-tailed random variables under the paradigm of multivariate regular variation possessing asymptotic tail independence in the upper tails and we study the average behavior of one of the variables given that the other one is large in an asymptotic sense. The presence of asymptotic tail independence might intuitively indicate that high values of one variable will have little influence on the expected behavior of the other; we observe that such a behavior is not always true. In fact, under a quite general set of conditions, we are able to calculate the asymptotic behavior of the expected value of a variable given that the other one is high.

A major application of assessing such a behavior is in terms of computing systemic risk, where one wants to assess risk contagion among two risk factors in a system. Proper quantification of systemic risk has been a topic of active research in the past few years; see [1, 3, 6, 15, 17, 28] for further details. Our study concentrates on two such measures of risk in a bivariate set-up where both factors are heavy-tailed and possess asymptotic tail independence. Note that our notion of risk contagion refers to the effect of one risk on another and vice versa. Risk contagion has

Email addresses: bikram@sutd.edu.sg (Bikramjit Das), vicky.fasen@kit.edu (Vicky Fasen-Hartmann)