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Asymptotic behavior of maximum likelihood estimators for a jump-type Heston model

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ABSTRACT

We study asymptotic properties of maximum likelihood estimators of drift parameters for a jump-type Heston model based on continuous time observations, where the jump process can be any purely non-Gaussian Lévy process of not necessarily bounded variation with a Lévy measure concentrated on $(-1, \infty)$. We prove strong consistency and asymptotic normality for all admissible parameter values except one, where we show only weak consistency and mixed normal (but non-normal) asymptotic behavior. It turns out that the volatility of the price process is a measurable function of the price process. We also present some numerical illustrations to confirm our results.

1. Introduction

Parameter estimation, especially studying asymptotic properties of maximum likelihood estimator (MLE) of drift parameters for Cox–Ingersoll–Ross (CIR) and Heston models is an active area of research mainly due to the wide range of applications of these models in financial mathematics.

The present paper gives a new contribution to the theory of asymptotic properties of MLE for jump-type Heston models based on continuous time observations. Concerning related works, due to the vast literature on parameter estimation for Heston models, we will restrict ourselves to mention only papers that investigate the very same types of questions. For a detailed and recent survey on parameter estimation for Heston models in general, see the Introduction of Barczy and Pap (2016).

Overbeck (1998) studied MLE of the drift parameters of the CIR process based on continuous time observations, which is also called square root process or Feller process. Ben Alaya and Kebaier (2012, 2013) made a progress in MLE for the CIR process, giving explicit forms of joint Laplace transforms of the building blocks of this MLE as well.

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The original Heston model (see Heston, 1993) takes the form

$$\begin{cases} \mathrm{d}Y_t = \kappa(\theta - Y_t)\,\mathrm{d}t + \sigma\sqrt{Y_t}\,\mathrm{d}W_t, \\ \mathrm{d}S_t = \mu S_t\,\mathrm{d}t + S_t\sqrt{Y_t}\big(\varrho\,\mathrm{d}W_t + \sqrt{1-\varrho^2}\,\mathrm{d}B_t\big), \end{cases} \qquad t \in [0, \infty), \tag{1.1}$$

where $(S_t)_{t\in[0,\infty)}$ is the price process of an asset, $\mu \in \mathbb{R}$ is the rate of return of the asset, $\theta \in (0, \infty)$ is the so-called long variance (long run average price variance, i.e., the limit of $\mathbb{E}(Y_t)$ as $t \to \infty$), $\kappa \in (0, \infty)$ is the rate at which $(Y_t)_{t\in[0,\infty)}$ reverts to θ, $\sigma \in (0, \infty)$ is the so-called volatility of the volatility and $\varrho \in (-1, 1)$ is the correlation between the driving standard Wiener processes $(W_t)_{t\in[0,\infty)}$ and $(\varrho W_t + \sqrt{1-\varrho^2 B_t})_{t\in[0,\infty)}$. Barczy and Pap (2016) investigated the Heston model with another parametrization.

In this paper we study a jump-type Heston model (also called a stochastic volatility with jumps model, SVJ model)

$$\begin{cases} \mathrm{d}Y_t = \kappa(\theta - Y_t)\,\mathrm{d}t + \sigma\sqrt{Y_t}\,\mathrm{d}W_t, \\ \mathrm{d}S_t = \mu S_t\,\mathrm{d}t + S_t\sqrt{Y_t}\big(\varrho\,\mathrm{d}W_t + \sqrt{1-\varrho^2}\,\mathrm{d}B_t\big) + S_{t-}\,\mathrm{d}L_t, \end{cases} \qquad t \in [0, \infty), \tag{1.2}$$

where $(L_t)_{t\in[0,\infty)}$ is a purely non-Gaussian Lévy process independent of $(W_t, B_t)_{t\in[0,\infty)}$ with Lévy–Khintchine representation

$$\mathbb{E}(\mathrm{e}^{\mathrm{i}uL_1}) = \exp\left\{\mathrm{i}\gamma u + \int_{-1}^{\infty}(\mathrm{e}^{\mathrm{i}uz} - 1 - \mathrm{i}uz\mathbb{1}_{[-1,1]}(z))\,m(\mathrm{d}z)\right\}, \qquad u \in \mathbb{R}, \tag{1.3}$$

where $\gamma \in \mathbb{R}$ and m is a Lévy measure concentrated on $(-1, \infty)$ with $m(\{0\}) = 0$. Here, let us recall that the Lévy process L has finite variation on each interval $[0, t]$, $t \in [0, \infty)$, if and only if $\int_{-1}^{1}|z|\,m(\mathrm{d}z) < \infty$, see, e.g., Sato (1999, Theorem 21.9). We point out that the assumption $\mathbb{P}(Y_0 \in [0, \infty), S_0 \in (0, \infty)) = 1$ and the assumption in question on the support of the Lévy measure m assure that $\mathbb{P}(S_t \in (0, \infty)$ for all $t \in [0, \infty)) = 1$ (see Proposition 2.1), so the process S can be used for modeling prices in a financial market. For a good survey on jump-type Heston models, pricing and hedging in these models, see Runggaldier (2003). In fact, the model (1.2) is quite popular in finance with the special choice of the Lévy process L as a compound Poisson process. Namely, let

$$L_t := \sum_{i=1}^{\pi_t}(\mathrm{e}^{J_i} - 1), \qquad t \in [0, \infty), \tag{1.4}$$

where $(\pi_t)_{t\in[0,\infty)}$ is a Poisson process with intensity $\lambda \in (0, \infty)$, $(J_i)_{i\in\mathbb{N}}$ is a sequence of independent identically distributed random variables having no atom at zero (i.e., $\mathbb{P}(J_1 = 0) = 0$), and being independent of π as well. We also suppose that π, $(J_i)_{i\in\mathbb{N}}$, W and B are independent. One can interpret J as the jump size of the logarithm of the asset price. Then

$$\begin{aligned} \mathbb{E}(\mathrm{e}^{\mathrm{i}uL_1}) &= \exp\left\{\lambda\int_{-1}^{\infty}(\mathrm{e}^{\mathrm{i}uz} - 1)\,m(\mathrm{d}z)\right\} \\ &= \exp\left\{\mathrm{i}u\lambda\int_{-1}^{1} z\,m(\mathrm{d}z) + \lambda\int_{-1}^{\infty}(\mathrm{e}^{\mathrm{i}uz} - 1 - \mathrm{i}uz\mathbb{1}_{[-1,1]}(z))\,m(\mathrm{d}z)\right\}, \qquad u \in \mathbb{R}, \end{aligned}$$

has the form (1.3) with m being the distribution of $\mathrm{e}^{J_1} - 1$ and $\gamma = \lambda\int_{-1}^{1} z\,m(\mathrm{d}z)$. Moreover, S_t takes the form

$$S_t = S_0\exp\left\{\int_0^t\left(\mu - \frac{1}{2}Y_u\right)\mathrm{d}u + \int_0^t\sqrt{Y_u}\,(\varrho\,\mathrm{d}W_u + \sqrt{1-\varrho^2}\,\mathrm{d}B_u) + \sum_{i=1}^{\pi_t}J_i\right\} \tag{1.5}$$

for $t \in [0, \infty)$, see (2.1). We note that the SDE (1.2) with the Lévy process L given in (1.4) has been studied, e.g., by Bates (1996, equation (1)), Bakshi et al. (1997, equations (1) and (2) with $R \equiv 0$), by Broadie and Kaya (2006, equations (30)–(31)) (where a factor S_{t-} is missing from the last term of equation (30)), by Runggaldier (2003, Remark 3.1, with $\lambda_t \equiv \lambda$) and by Sun et al. (2017, equation 1 with $J_v = 0$). Bates (1996), Bakshi et al. (1997) and Broadie and Kaya (2006) have chosen the common distribution of J as a normal distribution. Bakshi et al. (1997) used this model for studying (European style) S&P 500 options, e.g., they derived a practically implementable closed-form pricing formula. Broadie and Kaya (2006) gave an exact simulation algorithm for this model, further, they considered the pricing of forward start options in this model. Sun et al. (2017) have chosen the common distribution of J as a normal distribution, a one-sided exponential distribution or a two-sided distribution, and they applied the Fourier–cosine series expansion method for pricing vanilla options under these jump-type Heston models.

The aim of this paper is to study the MLE of the parameter $\boldsymbol{\psi} := (\theta, \kappa, \mu)$ for the model (1.2) based on continuous time observations $(Y_t, S_t)_{t\in[0,T]}$ with $T \in (0, \infty)$, starting the process (Y, S) from some deterministic initial value $(y_0, s_0) \in (0, \infty)^2$ supposing that σ, ϱ, γ and the Lévy measure m are known. Here we stress that under these assumptions, the underlying statistical space corresponding to the parameters $(\kappa, \theta, \mu) \in (0, \infty)^2 \times \mathbb{R}$ is identifiable, however it would not be true for the statistical space corresponding to the parameters $(\kappa, \theta, \mu, \gamma) \in (0, \infty)^2 \times \mathbb{R}^2$. We call the attention that the MLE in question contains stochastic integrals with respect to L. We prove that, for all $t \in [0, T]$, L_t is a measurable function (i.e., a statistic) of $(S_t)_{t\in[0,T]}$, by providing a sequence of measurable functions of $(S_t)_{t\in[0,T]}$ converging in probability to L_t, see Remark 2.4 (note that this sequence depends on γ and m as well). Further, it turns out that Y_t for all $t \in [0, T]$, and the parameters σ and ϱ are