Statistical Methodology 33 (2016) 83–95

Contents lists available at ScienceDirect

Statistical Methodology

journal homepage: www.elsevier.com/locate/stamet

Estimation and goodness-of-fit in latent trait models: A comparison among theoretical approaches

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ARTICLE INFO

Article history:
Received 24 July 2015
Received in revised form
25 April 2016
Accepted 11 May 2016
Available online 26 May 2016

Keywords:
Ordinal data
Item response theory approach
Latent trait models
Simulation
Goodness-of-fit indices

ABSTRACT

Two theoretical approaches are usually employed for the fitting of ordinal data: the underlying variables approach (UV) and the item response theory (IRT). In the UV approach, limited information methods [generalized least squares (GLS) and weighted least squares (WLS)] are employed. In the IRT approach, fitting is carried out with full information methods [Proportional Odds Model (POM), and the Normal Ogive (NOR)]. The four estimation methods (GLS, WLS, POM and NOR) are compared in this article at the same time, using a simulation study and analyzing the goodness-of-fit indices obtained. The parameters used in the Monte Carlo simulation arise from the application of a political action scale whose two-factor structure is well known. The results show that the estimation method employed affects the goodness-of-fit to the model. In our case, the IRT approach shows a better fitting than UV, especially with the POM method.

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http://dx.doi.org/10.1016/j.stamet.2016.05.002
1572-3127/

1. Introduction

A theory of latent traits supposes that in a test situation, examinee performance on a test can be predicted (or explained) by defining characteristics of examinees, referred to as traits, estimating scores for examinees on these traits, and using the scores to predict or explain test performance [19]. Since the traits are not directly measurable and therefore "unobservable", they are referred to as latent variables. These variables can only be indirectly studied through their multiple manifestations, and they are known in the academic literature as latent trait variables. A common example of latent trait variables is the depression. It can be said that a person suffers from a depression by the observation of his behavior; however most academics would say that depression is not equivalent to the behavior observed. Depression is an intangible phenomena that influences behavior, but it is not directly observed and measured, it is a latent phenomenon [18].

In the analysis of data in latent trait models usually consider ordinal variables in the process of measuring. Typical ordinal variables expose attitudinal statements with alternative answers like "strongly disagree", "disagree", "strongly agree" or "very dissatisfied", "dissatisfied", "satisfied" and "very satisfied" [14]. The treatment of the ordinal data in latent trait models is usually carried out following two approaches: The underlying variables (UV) and the item response theory (IRT) [23,6,28,33].

The underlying variables approach (UV) assumes that the variables are *fictitious continuous* since they are neither observed nor latent variables [6]. The estimation methods used in this approach are limited information methods and use either the polychoric correlation matrix, or the asymptotic covariance matrix as data input for the estimation of the parameters [32,21,22]. The procedure for such estimation comprises three steps: The first one is the first order statistics such as thresholds, means and variances and they are estimated by maximum likelihood [32,21,22]. In the second stage, second order statistics such as polychoric correlations are estimated by conditional maximum likelihood for given first stage estimates [23]. The third step is the estimation of model parameters (factorial weights): Muthén [32] uses the generalized least squares (GLS) and Jöreskog [21] uses the weighted least squares (WLS) from the asymptotic covariance matrix. The GLS despite assuming multivariate normality, can be used with non-normal data, employed the polychoric correlation matrix [21,17]. Methods based on the asymptotic distribution free of data [13], like WLS, are also used in non-normal conditions, although WLS shows limitations regarding the sample size and the number of variables.

The Item Response Theory (IRT) approaches assume that posits a nonlinear relationship between the underlying/latent construct and the observed score at the ítem/subscale level [34]. The methods used are full information and logistic function since the Proportional Odds Model (POM), and the Normal Ogive (NOR) [36,37,30,31]. These procedures have the advantage of assuming different to normal data distributions and the analysis unit is the answer patterns of the subjects, avoiding information losses [27]. In addition, the IRT approach is less restrictive regarding the number of variables, although the computational cost is higher when many variables are used [8,6,1,33,5].

In the academic literature there are several studies that compare these two approaches. Jöreskog and Moustaki [23] use simulated data for the comparison between limited information estimation methods such as the underlying bivariate normal and complete information methods, such as the multivariate normal method, POM and NOR. Maydeu-Olivares [26], in an attempt to establish a unified theoretical framework, compares the parameter estimation in linear and nonlinear IRT approaches with the factor analysis, using maximum likelihood and unweighted least squares with limited and complete information. Moustaki, Jöreskog, and Mavridis [29] compare POM and NOR methods of complete information with the robust maximum likelihood and with the WLS of limited information; emphasizing the covariate effects on the latent variables. Other researches that seek to compare the two approaches can be found in [7,14,34].

All these studies focus on the parameter comparison, studying the accuracy of estimates, but they seldom study the effects of the estimation method used in fitting latent trait models. In this regard, the aim of this work is to show how goodness-of-fit is affected by the choice of estimation method (GLS, WLS, NOR and POM), using ordinal data. Consequently, this paper attempts to develop a framework of analysis that allows researchers and practitioners to discern how to choose an estimation method among the available approaches to analyzing ordinal data. This would allow the analysis of aspects