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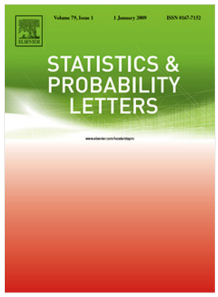

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Weak and One-sided Strong Laws for Random Variables with Infinite Mean

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Abstract

Let a_j be positive weight constants and X_j be independent non-negative random variables $(j = 1, 2, \ldots)$ and $S_n(\mathbf{a}) = \sum_{i=1}^n a_i X_i$. If the X_j have the same relatively stable distribution, then under mild conditions there exist constants $b_n \to \infty$ such that $\overline{W}_n(\mathbf{a}) = b_n^{-1} S_n(\mathbf{a}) \xrightarrow{p} 1$, i.e., a weak law of large numbers holds. If the weights comprise a regularly varying sequence, then under some additional technical conditions, this outcome can be strengthened to a strong law if and only if the index of regular variation is -1. This paper addresses a case where the X_j are not identically distributed, but rather the tail probability $P(a_j X_j > x)$ is asymptotically proportional to $a_j(1 - F(x))$, where F is a relatively stable distribution function. Here the weak law holds but the strong law does not: under typical conditions almost surely $\liminf_{n\to\infty} \overline{W}_n(\mathbf{a}) = 1$ and $\limsup_{n\to\infty} \overline{W}_n(\mathbf{a}) = \infty$.

Keywords: Relative stability; Weighted laws of large numbers; Regular variation

AMS 2000 Subject Classifications: 60F05, 60F15

1 Introduction

Let $\mathbf{a} = (a_j : j = 1, 2, \ldots)$ be a sequence of positive weights, $(X_j : j = 1, 2, \ldots)$ a sequence of independent non-negative random variables and let $S_n(\mathbf{a}) = \sum_{i=1}^n a_i X_i$. If the X_j are identically distributed, then their common distribution function (DF) F is called relatively stable (and written $F \in RS$) if there are norming constants $b_n > 0$ such that $b_n^{-1} \sum_{i=1}^n X_i \xrightarrow{p} 1$ (Bingham et al. (1987, p. 372)). When $F \in RS$ and there are (norming) constants b_n and weights $a_n > 0$ such that $b_n^{-1} \max_{i\le n} a_i \to 0$ (implying that $b_n \to \infty$), Adler and Pakes (2017) give a necessary and sufficient condition for the weak law of large numbers $\overline{W}_n(\mathbf{a}) := S_n(\mathbf{a})/b_n \xrightarrow{p} 1$. Further, assuming that the weight sequence is regularly varying, they show that the weak law extends

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