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GRADIENT AND STABILITY ESTIMATES OF HEAT KERNELS FOR FRACTIONAL POWERS OF ELLIPTIC OPERATOR

YONG CHEN, YAOZHONG HU, AND ZHI WANG

ABSTRACT. Gradient and stability type estimates of heat kernel associated with fractional power of a uniformly elliptic operator are obtained. L^p -operator norm of semigroups associated with fractional power of two uniformly elliptic operators are also obtained.

Keywords. Gradient Estimates, Stability, Subordination, Fractional Powers.

MSC(2010): 60J35, 47D07.

1. INTRODUCTION AND MAIN CONCLUSIONS

Let \mathbf{D} be a domain in \mathbb{R}^d and let $a : \mathbb{R}^d \rightarrow \mathbb{R}^{d^2}$ be a matrix valued function with C^β or measurable entries. The operator $H = \nabla(a(x)\nabla)$ generated a semigroup P_t which is given by $P_t f(x) = \int_{\mathbb{R}^d} p_t(x, y) f(y) dy$. Heat kernel, gradient and stability estimates associated with this semigroup are well-studied (see [2], [3], [10]). In this paper we are concerned with the similar estimates for the semigroup generated by the fractional powers of H , namely, $Q_t = e^{-t(-H)^\alpha}$, where $\alpha \in (0, 1)$ will be fixed throughout this paper. Our motivation is recent works on fractional diffusion in random environment (see [1, 6] and references therein) arisen from super and sub diffusion in random environment. However, we shall deal with this problem in separate project.

First let us recall a result. Using the classical Bromwich contour integral, Pollard in [7] obtained the following formula for the inverse Laplace transform of the function e^{-u^α} .

$$e^{-u^\alpha} = \int_0^\infty e^{-us} g(\alpha, s) ds, \quad u \geq 0. \quad (1.1)$$

where

$$g(\alpha, s) = \frac{1}{\pi} \int_0^\infty e^{-su} e^{-u^\alpha \cos \pi \alpha} \sin(u^\alpha \sin \pi \alpha) du, \quad s \geq 0. \quad (1.2)$$

is a probability density function of $s \geq 0$. This class of density functions $g(\alpha, s)$ is called strictly α -stable law which plays important role in the theory of probability.

Denote

$$\lambda_t(ds) = g_t(\alpha, s) ds := t^{-\frac{1}{\alpha}} g(\alpha, t^{-\frac{1}{\alpha}} s) ds. \quad (1.3)$$

Then

$$\begin{aligned} e^{-u^\alpha t} &= e^{-(ut^{1/\alpha})^\alpha} = \int_0^\infty e^{-ut^{1/\alpha}s} g(\alpha, s) ds \\ &= \int_0^\infty e^{-us} g_t(\alpha, s) ds \end{aligned}$$

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