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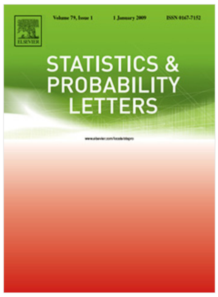

Gradient and stability estimates of heat kernels for fractional powers of elliptic operator

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GRADIENT AND STABILITY ESTIMATES OF HEAT KERNELS FOR FRACTIONAL POWERS OF ELLIPTIC OPERATOR

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Abstract. Gradient and stability type estimates of heat kernel associated with fractional power of a uniformly elliptic operator are obtained. L^p-operator norm of semigroups associated with fractional power of two uniformly elliptic operators are also obtained.

Keywords. Gradient Estimates, Stability, Subordination, Fractional Powers.
MSC(2010): 60J35, 47D07.

1. Introduction and main conclusions

Let $\mathbf{D}$ be a domain in $\mathbb{R}^d$ and let $a : \mathbb{R}^d \to \mathbb{R}^{d^2}$ be a matrix valued function with C^β or measurable entries. The operator $H = \nabla(a(x)\nabla)$ generated a semigroup P_t which is given by $P_t f(x) = \int_{\mathbb{R}^d} p_t(x, y) f(y) dy$. Heat kernel, gradient and stability estimates associated with this semigroup are well-studied (see [2], [3], [10]). In this paper we are concerned with the similar estimates for the semigroup generated by the fractional powers of H, namely, $Q_t = e^{-t(-H)^\alpha}$, where $\alpha \in (0, 1)$ will be fixed throughout this paper. Our motivation is recent works on fractional diffusion in random environment (see [1, 6] and references therein) arisen from super and sub diffusion in random environment. However, we shall deal with this problem in separate project.

First let us recall a result. Using the classical Bromwich contour integral, Pollard in [7] obtained the following formula for the inverse Laplace transform of the function e^{-u^α}.

$$e^{-u^\alpha} = \int_0^\infty e^{-us} g(\alpha,\, s) \mathrm{d}s, \qquad u \geq 0. \tag{1.1}$$

where

$$g(\alpha,\, s) = \frac{1}{\pi} \int_0^\infty e^{-su} e^{-u^\alpha \cos \pi\alpha} \sin(u^\alpha \sin \pi\alpha)\, \mathrm{d}u, \quad s \geq 0. \tag{1.2}$$

is a probability density function of $s \geq 0$. This class of density functions $g(\alpha, s)$ is called strictly α-stable law which plays important role in the theory of probability.

Denote

$$\lambda_t(ds) = g_t(\alpha,\, s) ds := t^{-\frac{1}{\alpha}} g(\alpha,\, t^{-\frac{1}{\alpha}} s) ds. \tag{1.3}$$

Then

$$\begin{aligned} e^{-u^\alpha t} &= e^{-(ut^{1/\alpha})^\alpha} = \int_0^\infty e^{-ut^{1/\alpha} s} g(\alpha,\, s) \mathrm{d}s \\ &= \int_0^\infty e^{-us} g_t(\alpha,\, s) \mathrm{d}s \end{aligned}$$