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Limit theory for the empirical extremogram of random fields

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Abstract

Regularly varying stochastic processes are able to model extremal dependence between process values at locations in random fields. We investigate the empirical extremogram as an estimator of dependence in the extremes. We provide conditions to ensure asymptotic normality of the empirical extremogram centred by a pre-asymptotic version. The proof relies on a CLT for exceedance variables. For max-stable processes with Fréchet margins we provide conditions such that the empirical extremogram centred by its true version is asymptotically normal. The results of this paper apply to a variety of spatial and space–time processes, and to time series models. We apply our results to max-moving average processes and Brown–Resnick processes.

MSC: primary 60F05; 60G70; 62G32; secondary 37A25; 62M30

Keywords: Brown–Resnick process; Empirical extremogram; Extremogram; Max-moving average process; Max-stable process; Random field; Spatial CLT; Spatial mixing

1. Introduction

The extremogram measures extremal dependence in a strictly stationary regularly varying stochastic process and can hence be seen as a correlogram for extreme events. It was introduced in Davis and Mikosch [10] for time series (also in Fasen et al. [17]), and they show consistency and asymptotic normality of an empirical extremogram under weak mixing conditions. Davis et

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al. [11] give a profound review of the estimation theory for time series with various examples. For a discussion of the role of the extremogram in dependence modelling of extremes we refer again to [10]. As it is spelt out there, it is the covariance function of indicator functions of exceedance events in an asymptotic sense. Also in that paper classical mixing conditions as presented in Ibragimov and Linnik [21], on which we rely in our work, are compared to the extreme mixing conditions D and D' often used in extreme value theory (cf. Embrechts et al. [16], Section 4.4, and Leadbetter et al. [23], Sections 3.1 and 3.2).

The extremogram and its empirical estimate have been formulated for spatial d-dimensional stochastic processes by Cho et al. [7] and for space–time processes in Buhl et al. [4] and Steinkohl [28], when observed on a regular grid. The extremogram is defined for strictly stationary regularly varying stochastic processes, where all finite-dimensional distributions are in the maximum domain of attraction of some Fréchet distribution. Among other results, based on the seminal paper [2] by Bolthausen, [7] proves a CLT for the empirical extremogram sampled at different spatial lags, centred by the so-called pre-asymptotic extremogram. Such results also compare with a CLT for sample space–time covariance estimators derived in Li et al. [24], also based on [2].

The pre-asymptotic extremogram can be replaced in the CLT by the true one, if a certain bias condition is satisfied; in particular, the difference between the pre-asymptotic and the true extremogram must vanish with the same rate as the one given in the CLT. However, for many processes the assumptions required in [7] are too restrictive to satisfy this bias condition. We explain this in detail and present two models which exactly fall into this class; the max-moving average process and the Brown–Resnick process. These two processes are max-stable with Fréchet margins.

In this paper, we prove a CLT for the empirical extremogram centred by the pre-asymptotic extremogram for strictly stationary regularly varying stochastic processes, which relies on weaker conditions than the CLT stated in [7]. Our proof also partly relies on Bolthausen's CLT for spatial processes in [2]; however, we make important modifications so that the bias condition mentioned above can be satisfied, and thus a CLT for the empirical extremogram centred by the true one for many more processes becomes possible. The proof is based on a big block/small block argument, similarly to [10].

Our interest is of course in a CLT centred by the true extremogram, and whether such a CLT is possible depends on the specific regularly varying process. If the process has finite-dimensional max-stable distributions, in our framework equivalent to having finite-dimensional Fréchet distributions, we can give conditions such that a CLT of that kind is possible. Here we need the weaker mixing conditions of our version of Bolthausen's CLT compared to [7]. Furthermore, under conditions such that a CLT centred by the true extremogram is not possible, a bias-corrected estimator can be defined, which we do in the accompanying paper Buhl et al. [4] for the Brown–Resnick process.

Our paper is organised as follows. In Section 2 we present the general model class of strictly stationary regularly varying processes in $\mathbb{R}^d$ for $d \in \mathbb{N}$. We also define here the extremogram for such processes. In Section 3 we define the empirical extremogram based on grid observations, and also the pre-asymptotic extremogram. Section 4 is devoted to the CLT for the empirical extremogram centred by the pre-asymptotic extremogram and to our examples of max-stable spatial processes; max-moving average processes and Brown–Resnick processes. We discuss in detail the problem of a CLT for the empirical extremogram and compare our new conditions for the CLT to hold with those in previous work, particularly with those given in [7]. For processes with Fréchet margins we prove a CLT for the empirical extremogram centred by the true extremogram. The proof of the CLT is given in Section 5.

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