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The recognition and the constitution of the theorems of closure

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Abstract

This papers analyzes how several geometric theorems, that were considered to be disconnected from each other in the beginning of the nineteenth century, have been progressively recognized as elements of a bigger whole called "the theorems of closure." In particular, we show that the constitution of this set of theorems was grounded on the use of encompassing words, as well as observations of analogies, and searches for unifying points of view. In the concluding remarks, we discuss the relevancy of the notion of "family resemblance" to describe the categorization process of the theorems of closure during the nineteenth century.

Résumé

Cet article analyse la manière dont plusieurs théorèmes géométriques qui étaient considérés comme déconnectés les uns des autres au début du dix-neuvième siècle ont été progressivement reconnus comme des éléments d'un tout plus grand appelé "théorèmes de clôture". Nous montrons en particulier que la constitution de cet ensemble de théorèmes s'est faite par un emploi d'étiquettes particulières, mais aussi par l'observation d'analogies et par des recherches de points de vue unificateurs. Dans les remarques de conclusion, nous discutons de la pertinence de la notion d'"air de famille" pour decrire le processus de catégorisation des théorèmes de clôture au cours du dix-neuvième siècle.

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Keywords: History of geometry; Closure theorems; Porisms; Categorization; Jacob Steiner; Adolf Hurwitz

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1. Introduction

1.1. A category of problems and theorems

In different ages and places, practitioners of mathematics have distinguished and forged many categories such as disciplinary rubrics, families of objects, and classes of problems.[1] Accordingly, many historical investigations have been conducted to understand the significance of such categories for the people who created or used them, the dynamics of the research concealed behind their labels, or their role in the development of particular parts of mathematics. For instance, having aimed attention at the corresponding entries in various encyclopedic publications, Christian Gilain (2010) explained the evolution of the status of "analysis" during the Enlightenment; in another paper, I tackled the organization of the knowledge linked to a family of objects called "geometrical equations" and the role of this family in the process of assimilation of substitution theory in the second half of the nineteenth century, (Lê, 2016); as for classes of problems, Renaud Chorlay (2010) delineated how the so-called "Cousin problems," coming from complex analysis, were involved in the emergence of sheaf theory in the course of the twentieth century.[2]

If the contents of such categories have often been investigated, such investigations rarely extend to the processes of their creation.[3] The present article proposes to tackle this question in the case of a category of geometric problems and theorems that have been called *Schliessungsprobleme* and *Schliessungssätze* by German-speaking mathematicians since the beginning of the 1870s.[4]

One emblematic example belonging to this category is the problem about polygons and conics that Jean-Victor Poncelet enunciated and solved in his 1822 *Traité des propriétés projectives des figures*, (Poncelet, 1822), and that was recognized in 1876 by the mathematician and historian of mathematics Max Simon as "the most famous of the problems of closure."[5] This problem can be formulated as follows: two conics being given, one starts from a point A on one of them and draws a tangent to the other one, which defines a new point B on the first conic. Continuing this way, one constructs a polygonal line $ABCD\dots$ The problem is then to determine if it is possible to obtain a closed polygonal line, thus yielding a polygon inscribed in the first conic and circumscribed about the other. Poncelet then proved the theorem that if it is possible to find one such polygon with, say, n sides, then infinitely many polygons with n sides exist, every point on the first conic being the starting point of a closed polygon (see Figure 1).[6]

Two general remarks should be made at this point. The first one bears upon the difference between problems and theorems, a difference which can obviously be seen in the existence of the two labels *Schliessungsprobleme* and *Schliessungssätze*, and which shaped my explanations in the preceding paragraph. It echoes the distinction inherited from Greek Antiquity: problems primarily link to constructions of objects

[1] Note that in this paper, the word "category" will never refer to its current technical mathematical meaning.

[2] Aside from these recent examples dealing with explicit categories, Karine Chemla (2009) proved the existence of classes of problems that were seen (but not made explicit) by commentators of the *Nine Chapters*. Further, the collective project conducted by Alain Bernard (2015) tackled the notion of "series of problems" to designate a specific textual genre characterized by sequences of mathematical questions and answers.

[3] See, however, the descriptions given in (Chorlay, 2010, 19–24, 65–66).

[4] I will translate *Schliessungsprobleme* and *Schliessungssätze* by "problems of closure" and "theorems of closure." The word *Schliessungstheoreme* was also used by mathematicians of the time instead of *Schliessungssätze*, and similarly, although much less frequently, *Schliessungsaufgaben* sometimes replaced *Schliessungsprobleme*. Following (Müller, 1900, 143, 260, 276, 293), I chose to adopt the same translation for *Theorem* and *Satz* on one hand, and for *Problem* and *Aufgabe* on the other hand.

[5] "[Das] bekanntest[e] der Schliessungsproblemen." (Simon, 1876, 303). As has been remarked in (Bos et al., 1987, 311–313), Poncelet proved this theorem already in 1813–1814, yet with different methods as those employed in the *Traité*. However, we will see that the authors who worked on the theorems of closure mostly cited Poncelet's 1822 book. On the *Traité*, see for instance (Friedelmeyer, 2011; Nabonnand, 2015).

[6] In this paper, all the figures whose caption do not contain a citation are mine.

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