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A footnote to The crisis in contemporary mathematics

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Abstract

We examine the preparation and context of the paper "The Crisis in Contemporary Mathematics" by Errett Bishop, published 1975 in *Historia Mathematica*. Bishop tried to moderate the differences between Hilbert and Brouwer with respect to the interpretation of logical connectives and quantifiers. He also commented on Robinson's *Non-standard Analysis*, fearing that it might lead to what he referred to as 'a debasement of meaning.' The 'debasement' comment can already be found in a draft version of Bishop's lecture, but not in the audio file of the actual lecture of 1974. We elucidate the context of the 'debasement' comment and its relation to Bishop's position vis-a-vis the Law of Excluded Middle.

MSC: primary 01A60; secondary 26E35, 03B20, 03F60

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1. Introduction

We will compare three extant versions of Errett Bishop's 1974 lecture entitled "The crisis in contemporary mathematics." Errett Bishop (1928–1983) delivered a plenary lecture in the session on the Foundations of Mathematics of the Workshop on the Evolution of Modern Mathematics organized by the American Academy of Arts and Sciences (AAAS) in 1974.

Three versions of the lecture are extant. The first one is a 2-page initial draft of the lecture [Bishop, 1974a]. The second is an audio recording [Bishop, 1974b] of the lecture delivered on 9 august 1974. The third is the published version of the lecture in *Historia Mathematica* [Bishop, 1975].

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2. The three versions

2.1. The draft version

The draft of the lecture already sets out its main theme, namely the Brouwer–Hilbert differences on the meaning of logical connectives and quantifiers:

> What I am recommending, and I do not know whether the possibility occured [sic] to Hilbert, is that we accept Brouwer's definitions of "or", "there exists", and all the other connectives and quantifiers, without damaging the paradise that Hilbert wished to preserve. [Bishop, 1974a, p. 1]

There follows a paragraph concerning the work of Abraham Robinson [Robinson, 1966] on infinitesimal analysis[1] and of H. Jerome Keisler [Keisler, 1971] on infinitesimal calculus:

> A more recent attempt at mathematics by formal finesse is non-standard analysis. I gather that it has met with some degree of success, whether at the expense of giving significantly less meaningful proofs I do not know. My interest in non-standard analysis is that attempts are being made to introduce it into calculus courses. It is difficult to believe that debasement of meaning could be carried so far. [Bishop, 1974a, p. 2]

Bishop goes on to discuss recursive function theory, and then comments on applications in science:

> The reason that mathematics is so successful in the physical sciences is not clear. To Hermann Weyl, the utility of mathematics extended even to that part of mathematics that was not inherently computational. Although I hesitate to disagree with such an authority, my own impression is that the opposite is true. It would be interesting and worthwhile to settle this point. [Bishop, 1974a, p. 2]

2.2. The published version

Bishop's lecture was published in *Historia Mathematica* in 1975 as part of the proceedings of the AAAS workshop. The 11-page published version [Bishop, 1975] contains an expanded discussion of Brouwer–Hilbert disagreements over connectives and quantifiers, followed by a constructive analysis of the classical result that a function of bounded variation is differentiable almost everywhere. Bishop's conclusion, echoing the corresponding remarks in the draft version, is the following:

> In a way, the imaginary dialogue that I presented here might be regarded as a historical investigation if you believe as I do that it shows how two titanic figures such as these might have reached an accommodation that would have changed the course of mathematics in a profound way, had they spoken to each other with less emotion and more concern for understanding each other. Instead, Hilbert tried to show that it was all right to neglect computational meaning, because it could ultimately be recovered by an elaborate formal analysis of the techniques of proof. This artificial program failed.[2] [Bishop, 1975, p. 513]

[1] For a historical analysis of the genesis of Robinson's theory see [Dauben, 1995].

[2] Bishop's negative appraisal notwithstanding, the proof mining program spearheaded by Ulrich Kohlenbach (see e.g., [Kohlenbach, 2008]) has been successful at extracting computational content from proofs in classical mathematics, going as far as improving known results in the literature (classical or constructive). The proof mining program is the archetype of an "elaborate formal analysis of the techniques of proof" which has successfully produced a plethora of natural results.

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