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Effects of natural frequency ratio on vortex-induced vibration of a cylindrical structure

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ABSTRACT

In this study, vortex-induced vibration (VIV) of a circular cylinder at the low Reynolds number of 200 is simulated by a transient coupled fluid–structure interaction numerical model. The transient coupling between the fluid and the structure is updated by on-line transmission between fluid dynamic loads and structure response data. The structural vibration of the cylinder influences the flow around it, and the change of fluid flow in turn influences the response of the cylinder. The boundary layer near the cylinder is updated by dynamic mesh technology with grid updating at each time step. This method successfully simulates the vortex generation and the real-time flow field of the cylinder. Considering VIV with low reduced damping parameters, the trend of the lift coefficient, the drag coefficient and the displacement of cylinder are analyzed under different oscillating frequencies of the cylinder. The frequency ratio α is a very important parameter, when α is small ($\alpha = 0.5$), the amplitude of lift coefficient of an elastic cylinder is large and the response of cylinder is weak. With an increase in α, the lateral displacement of the cylinder increases, but the amplitude of lift coefficient decreases. The amplitude of the lift coefficient reaches its minimum value when α is between 0.9 and 1.2. After that, the amplitude of the lift coefficient begins to increase. The typical nonlinear phenomena of locked-in, beat and phase-switch can be captured successfully. The evolution of vortex shedding from the cylinder with varied α is discussed. The trajectory of the two degrees of freedom (2 DOF) case at different α is also discussed; all appear to have a "Fig. 8" shape. A fast Fourier transformation technique is used to obtain the frequency characteristics of the cylindrical structure vibration. Using the 2 DOF cylinder model in place of the one degree of freedom (1 DOF) cylinder model presents several advantages in simulating the nonlinear characteristics of cylindrical structures including the capacity to model the crosswise vibration generated by in-line vibration.

1. Introduction

The flow-induced vibration of a cylindrical structure is a very common problem, especially in maritime environments such as undersea pipelines, offshore risers, and cables, which has arouse the many scholar extensively study in recent decades. The research of VIV for the design of the marine structure is very important [1]. The VIV is a typical fluid–structure interaction problem. The fluid force generated by the vortex around the riser makes the riser vibrated; on the other hand, the oscillating riser also affects the flow field around it. The general process of fluid–structure interaction is like this.

Early VIV research mainly is the study of the one degree of freedom (1 DOF) vibration of a circular cylinder, that is to say, only allow cylindrical cylinder transverse vibration. Feng [2], Brika and Laneville [3,4], Anagnostopoulos [5], Khalak and Williamson [6] are just a few of the contributing researchers. The content of the vortex-induced vibration for different aspects have been well documented in several overviews, such as those of Sarpkaya [7], Williamson and Govardhan [8,9], Bearman [10], Okajima [11], Wu et al. [12]. However, the study of VIV considering the two degrees of freedom is not as much as the one degree of freedom. Research into the 2 DOF VIV of a circular cylinder is relatively rare. Jeon and Gharib [13] studied the vortex wake for the 1 and 2 DOF VIV of a circular cylinder, and reported that even a small streamwise motion can inhibited the formation of 2P (two pair) vortex.

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Nomenclature

$m^* = m/m_d$	ratio of oscillating mass over displaced mass
$\zeta = \delta/2\pi$	damping ratio
$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m+m_a}}$	natural frequency in still water
$m_a = \rho\pi D^2/4$	added mass
$U^* = U/(f_n D)$	reduced velocity in water
f_s	Strouhal frequency
$St = f_s D/U$	Strouhal
$Re = UD/\nu$	Reynolds number
$\alpha = f_n/f_0$	frequency ratio

Jauvtis and Williamson [14] studied the 2 DOF VIV of a circular cylinder at low mass ratio and reported a new response branch, the "super-upper" branch, which occurred when the mass ratios were reduced below $m^* = 6$. In the "super-upper" branch, the transverse amplitudes of the cylinder vibration can be 1.5 times of the cylinder diameter. Guilmineau and Queutey [15] studied the fluid around an elastically mounted rigid cylinder which is only allowed transverse vibration with low mass-damping. Vortex shedding around the cylinder was investigated numerically by the SST $k-\omega$ model. Dahl et al. [16] conducted 2 DOF tests on an elastically mounted rigid cylinder which is allowed transverse and stream-wise vibration at Re = 11,000–60,000. In their experiment, the mass ratios were less than 6.0, and the natural frequency ratios of the in-line to transverse varied from 1.0 to 1.9. They reported that the maximum transverse amplitude exceeded 1.35*D* (where *D* is the cylinder diameter), whereas the stream-wise response reached 0.6*D*. When the cylinder was in the largest amplitude response, the cylinder moved along a crescent-shaped orbital trajectory. Placzek et al. [17] reported the result on the cylinders forced or free oscillating in low Reynolds number flow and analyzed the vortex shedding modes related to the frequency response. Bahmani and Akbari [18] investigated the basic characteristics of the dynamic response and vortex shedding on an elastically mounted circular cylinder in laminar flow. Bao et al. [19] reported that the VIV of the isolated and tandem elastically mounted cylinders which are allowed 2DOF vibration at a series of the natural frequency ratio of the in-line to transverse. Kang and Jia [20] use the experiment to investigate 1 DOF and 2 DOF VIV of a cylinder. A "double peak" phenomenon was found within the range of the reduced velocities tested, moreover, a "2T" wake appeared in the vicinity of the second peak in the 2 DOF VIV experiment, and the trajectory of cylinder exhibited a reverse "C" shape, i.e., a "new moon" shape.

Despite the large number of studies [2–6,17,18] dedicated to the VIV of a cylinder which is only allowed to vibrate in the transverse direction, but there is a little research [13,14,16,19,20] that also allows the cylinder to vibrate in the in-line direction, with some researchers studying the law of synchronization and the amplitude of vibration in the 2 DOF case [13,16,20] and some researchers investigating the regime of synchronization, amplitude, and wake pattern of the isolated and tandem elastically mounted cylinders which are allowed 2DOF vibration at a series of the natural frequency ratio of the in-line to transverse [19]. However, the vortex pattern, trajectory, frequency characteristics and so on have not been investigated with varying frequency ratios $\alpha = f_n/f_0$. Moreover, the VIV of an elastic cylinder has a strongly nonlinear quality. There have been a few nonlinear analyses of phenomena such as locked-in, beat, and phase-switch. The frequency characteristics of the vibration of a cylindrical structure in 1 and 2 DOF cases have not been analyzed with varying frequency ratios $\alpha = f_n/f_0$. To the best of our knowledge, there is little discussion of the nonlinear phenomena of bifurcation in VIV or of the difference in nonlinear phenomena between VIV with 1 and 2 DOF.

The present work is to study 2 DOF VIV of an elastically mounted cylinder for the natural frequency ratio. The response, hydrodynamic forces, vortex shedding modes and trajectory of the cylinder will be systematically analyzed by comparing with 1 DOF case. The nonlinear phenomena such as "lock-in", "phase-switch", "beat" are analyzed at different natural frequency ratio, and find out the critical point of each nonlinear phenomenon. Finally, the frequency characteristics of the elastically mounted cylinder is also analyzed at different natural frequency ratio.

2. Governing equations and transient dynamic analysis

2.1. Governing equations

The two-dimensional, incompressible, Navier–Stokes equations in the Cartesian coordinate, which can be written as follows:

$$\nabla \cdot (\rho\vec{v}) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho\vec{v}) + \nabla \cdot (\rho\vec{v}\vec{v}) = -\nabla p + \nabla \cdot (\tau) + \vec{F} \tag{2}$$

where p is the static pressure, τ is the stress tensor (described below), and $\vec{F}$ are the external body forces. $\vec{v}$ is the velocity vector. For the two-dimensional, incompressible, Navier–Stokes equations, $\vec{v} = [u, v]$. The large eddy simulation (LES) method is employed for the solution of Eq. (2).

The stress tensor τ is given by

$$\tau = \mu(\nabla\vec{v} + \nabla\vec{v}^T) \tag{3}$$

where μ is the molecular viscosity.

The motion of the cylinder can be described by the dimensionless equations:

$$\ddot{u}_x + 2\zeta f_n \dot{u}_x + f_n^2 u_x = F_d/m \tag{4}$$

$$\ddot{u}_y + 2\zeta f_n \dot{u}_y + f_n^2 u_y = F_l/m \tag{5}$$

where u_x and u_y are the displacement of the cylinder in the x and y direction, respectively; ζ is the damping factor of the spring-damper-mass system and is set to 0.01; f_n is the natural frequency of the cylinder in still water; F_d and F_l are the drag and lift forces of the cylinder respectively; and m is the mass of the cylinder.

The drag coefficient and lift coefficient reflect the force on the cylinder, which are described as follows:

$$C_d = \frac{2F_d}{\rho U^2 D} \tag{6}$$

$$C_l = \frac{2F_l}{\rho U^2 D} \tag{7}$$

where U is the inlet stream-wise velocity and D is the diameter of the cylinder.

2.2. Transient dynamic theory

Transient dynamic analysis (sometimes called time-history analysis) is a technique used to determine the dynamic response