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Enhancing buckling capacity of a rectangular plate under uniaxial compression by utilizing an auxetic material

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Abstract Auxetic materials have previously been shown to enhance various performances due to its unusual property of becoming fatter when uniaxially stretched and thinner when uniaxially compressed (i.e., the materials exhibit a negative Poisson's ratio). The current study focuses on assessing the potential of an auxetic material to enhance the buckling capacity of a rectangular plate under uniaxial compression. The in-plane translational restraint along the unloaded edges that was often neglected in open literature is taken into consideration in our buckling model proposed in this study. The closed-form expressions for the critical buckling coefficient of the rectangle are provided and the predicted results agree well with those determined by the finite element method. Furthermore, the results indicate that the buckling performance of a rectangular plate under uniaxial compression can be significantly improved by replacing the traditional material that has a positive Poisson's ratio with an auxetic material when there is in-plane translation restraint along the unloaded edges.

1. Introduction

The Poisson's ratio defines the ratio between the transverse and axial strain in a load material.[1] Moreover, it is considered as an important material parameter that directly affects the mechanical properties of a structure. Most materials have Poisson's ratio values that range between 0 and 0.5; however, some materials, known as auxetic materials, display a negative Poisson's ratio. Auxetic materials behave contrary to what is expected. For example, when subjected to an axial tensile load, their transverse dimension increases. Furthermore, the counter-intuitive properties of auxetic materials lead to structures that also exhibit enhanced mechanical and other physical performances. Lakes[2,3] was the first to manufacture a novel foam structure with a negative Poisson's ratio of −0.7, and presently, the main focuses of this field are discovering new auxetic materials and novel applications for them.

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At the present, a large number of auxetic materials have been determined and manufactured. These materials encompass nearly all of the classes of materials, including polymers, composites, metals, and ceramics. For a detailed introduction, review literatures[4–7] can be examined. Furthermore, although natural auxetic materials exist, most auxetic materials are artificial. In addition, topology optimization can be employed to tailor special auxetic materials according to practical requirements,[8,9] and the results obtained by topology optimization can be directly manufactured by 3D printing technology (also called additive manufacturing).[10,11] Another popular topic in this field is the exploration of potential applications for autextic materials. The existing research results reveal that auxetic materials exhibit a higher resistance to indentation, shear resistance,[12] fracture resistance,[13] acoustic absorption,[14] damping,[15] energy absorption,[16] a wider band gap with lower frequency,[17] and so on.

Buckling is a common failure mode in the aerospace structure. How to improve the stability of structure becomes an attractive problem. Recently, the stability of auxetic materials has also received significant attention. One such instance was an investigation by Spadoni et al.[18] of the buckling behavior of a chiral cellular structure with a negative Poisson's ratio under flat-wise compression . Additionally, the global buckling behavior of auxetic cellular tubes based on inverted hexagonal honeycombs has been discussed.[19,20] The results clearly indicate that the use of auxetic structures can significantly improve (or can result in a significant improvement on) the buckling behavior as compared to similar non-auxetic arrangements. Moreover, Lim[21–23] discussed the potential applications of the auxetic plate and shell, and the buckling behavior of rectangular and circular thick auxetic plates were investigated. From this investigation, a highly accurate shear correction factor in terms of a Poisson's ratio from −1 to 0.5 was obtained. However, the effect of the unusual deformation mechanism of auxetic materials on buckling behavior remains unexplored.

The purpose of this paper is to enhance the buckling performance of a rectangular plate by replacing the traditional material with a positive Poisson's ratio with an auxetic material. The structure of this paper is as follows: first, the mechanism of the enhanced buckling for the rectangular auxetic plate is provided in Section 2. Next, the critical buckling coefficient of the rectangular plate elastically restrained against in-plane translation under uniaxial compression is determined in Section 3. Section 4 introduces the results and discussion, and finally, Section 5 gives the conclusion.

2. Mechanism of enhanced buckling performance for rectangular auxetic plate

Fig. 1 shows a simply supported rectangular thin plate of dimension $a \times b$ under biaxial compressive loads. The magnitude of the compressive load is N_0 at the edges $x = 0, a$. Likewise, it is γN_0 at the edges $y = 0, b$. For a rectangular isotropic plate, the buckling load under biaxial loading can be expressed as[24]

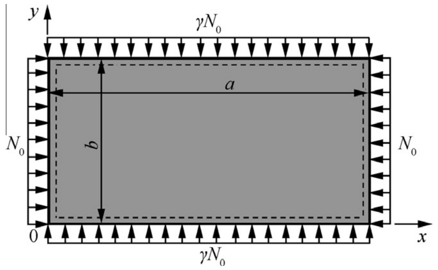

Fig. 1 Plate subjected to uniform compression along x and y directions.

$$N_0 = \frac{\pi^2 E t^3}{12b^2(1-v^2)} \cdot \frac{(m^2/\beta^2+1)^2}{m^2/\beta^2+\gamma} \tag{1}$$

where $\beta = a/b$ is the plate aspect ratio, E the elastic modulus of materials, v the Poisson's ratio, t the thickness of the plate, m the number of half waves along the x direction and y direction respectively. If the rotational constraint is full at the edges $y = 0$, b, the buckling load under biaxial loading can be obtained through the following expression

$$N_0 = \frac{\pi^2 E t^3}{12b^2(1-v^2)} \cdot \frac{(m^2/\beta^2 + 5.143\beta^2/m^2 + 2.472)}{1 + 1.236\gamma\beta^2/m^2} \tag{2}$$

A simply support square thin plate ($\beta = 1$) is selected as an example to demonstrate the mechanics of enhanced buckling performance for the rectangular auxetic plate. The critical buckling load subjected to a uniform compressive load N_0 on edges $x = 0$ and a (i.e. when $\gamma = 0$) can be calculated using Eq. (1)

$$N_{cr} = \frac{4\pi^2 E t^3}{12b^2(1-v^2)} \tag{3}$$

If the same uniform compressive load ($\gamma = 1$) is also applied along the y direction, the critical buckling load will decrease by 50%

$$N_{cr} = \frac{4\pi^2 E t^3}{12b^2(1-v^2)} \tag{4}$$

In contrast, the critical buckling load will increase by 75% when a half uniform tensile load ($\gamma = -0.5$) is applied along the y direction

$$N_{cr} = \frac{7.1429\pi^2 E t^3}{12b^2(1-v^2)} \tag{5}$$

In other words, the tensile load along the x direction is beneficial for improving the critical buckling load for a square plate when the compressive load is applied in the y direction.

Due to the Poisson's ratio effect, the plate becomes fatter when a material with a positive Poisson's ratio is used and thinner when a material with a negative Poisson's ratio is used. If the unloaded edges were to be subjected to elastic restraint against in-plane translation, then the induced equivalent load along the unloaded direction will be compressive for the material with a positive Poisson's ratio and will be tensile for the plate with a negative Poisson's ratio. Combined with the previous analysis, it can be predicted that an auxetic plate under uniaxial compression has a higher critical buckling load than one using a positive Poisson's ratio material when the unloaded edges are subjected to the elastic restraint against in-plane translation.

It is well-known that in practice, the ideal free boundary conditions for simply-supported or clamped plate never occur, and therefore translational restraint exists. Furthermore, while