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Research paper

Local and global existence of mild solution to impulsive fractional semilinear integro-differential equation with noncompact semigroup☆

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ABSTRACT

In this paper, we study local and global existence of mild solution for an impulsive fractional functional integro differential equation with non-compact semi-group in Banach spaces. We establish a general framework to find the mild solutions for impulsive fractional integro-differential equations, which will provide an effective way to deal with such problems. The theorems proved in this paper improve and extend some related conclusions on this topic. Finally, two applications are given to illustrate that our results are valuable.

1. Introduction

Fractional differential equations arise in many engineering and scientific disciplines as the mathematical modelling of systems and processes in the fields of physics, chemistry, aerodynamics, electrodynamics of complex medium, polymer rheology, and so forth, involves derivatives of fractional order. In consequence, the subject of fractional differential equations is gaining much importance and attention. For more details, see [1–7,16] and the references therein.

Impulsive differential equations have become important in recent years as mathematical models of phenomena in both physical and social sciences. There has been a significant development in impulsive theory especially in the area of impulsive differential equations with fixed moments. In [8], Heard and Rakin considered the following integro-differential equation in a Banach space X:

$$\begin{cases} u' + A(t)u(t) = f(t, u(t)) + \int_{t_0}^{t} q(s-t)g(s, u(s))ds, & t > t_0 \geq 0, \\ u(t_0) = u_0, \end{cases}$$

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where for each $t \geq 0$, the linear operator $-A(t)$ is the infinitesimal generator of an analytic semi-group in X, the nonlinear operator f is defined from $[0, \infty) \times X$ into X and satisfies a Hölder condition of the form

$$\|f(t, y_1) - f(s, y_2)\| \leq C[|t-s|^{\nu} + \|y_1 - y_2\|_k^{\gamma}],$$

$0 < \nu, k, \gamma < 1$, $\|\cdot\|$ is the norm on X and $\|\cdot\|_k$ is the graph norm on $X_k = D(A^k(0))$, the nonlinear map g is assumed to satisfy a local Lipschitz condition with respect to the norm of X. Also, the uniqueness of solution is proved under the restriction that the space X is a Hilbert space and $\gamma = 1$. In paper [9], Xiaobao Shu study the existence of mild solutions to a class of fractional semilinear integro-differential equation of order $1 < \alpha < 2$ in a Banach space X

$$\begin{cases} D_t^{\alpha} u(t) = Au(t) + f(t, u(t)) + \int_0^t q(t-s) g(s, u(s)) ds, & t \in [0, T], \\ u(0) + m(u) = u_0 \in X, \quad u'(0) + n(u) = u_1 \in X, \end{cases}$$

where $-A$ is a sectorial operator of type (M, θ, α, μ), defined from the domain $D(A) \subset X$ into X, the nonlinear map f, g are defined from $[0, T] \times X \to X$ are continuous functions.

In particular, the following fractional order integro-differential equation in a Banach space X:

$$\begin{cases} u^{\alpha}(t) + Au(t) = f(t, u(t)) + \int_0^t q(t-s) g(s, u(s)) ds, & t > 0, \quad \alpha \in (0, 1], \\ u(0) = u_0 \in X, \\ \Delta u|_{t=t_k} = I_k(u(t_k^-)), & k = 1, \ldots, m, \end{cases}$$

was mentioned and established the local and global existence of mild solutions using the fixed point technique by Rashid and Al-Omari in [10]. Here $-A$ is assumed to be an infinitesimal generator of a compact semi-group $T(t)$, $t \geq 0$. Due to the latest development regarding the mild solution for the considered problem of fractional order with impulsive conditions the definition of mild solution defined by the authors of [10] is not appropriate, because classical solutions of the impulsive fractional differential equations do not satisfy the definition of a mild solution given by the author and the semi-group property $T(t+s) = T(t)T(s)$ for the system is not used correctly, for more details (see [11,12]).

However, to the best of our knowledge, local and global existence of mild solution for an impulsive fractional functional integro differential equation with non-compact semi-group in Banach spaces has not been investigated yet. Motivated by this consideration, in this paper, we investigate local and global existence of mild solution for an impulsive fractional functional integro differential equation with non-compact semi-group in Banach spaces X:

$$\begin{cases} {}^cD_t^{\alpha} u(t) + Au(t) = f(t, u(t)) + \int_0^t q(t-s) g(s, u(s)) ds, & t \geq 0, t \neq t_k, \\ \Delta u|_{t=t_k} = I_k(u(t_k^-)), & k = 1, \ldots, m, \\ u(0) = u_0 \in X, \end{cases} \tag{1}$$

where ${}^cD_t^{\alpha}$ is the Caputo fractional derivative of order $\alpha \in (0, 1]$, A: $D(A) \subset X \to X$ is a closed linear operator and $-A$ generates a uniformly bounded C_0-semi-group $T(t)(t \geq 0)$ in X, the nonlinear maps f, g: $[0, \infty) \times X \to X$, and q: $I \to X$, are continuous. $I = [0, T)$, $0 < T \leq \infty$, $u_0 \in X$. $I_k : X \to X$, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = T$, $\Delta u|_{t=t_k} = u(t_k^+) - u(t_k^-)$, $u(t_k^+) = \lim_{m \to 0} u(t_k + m)$ and $u(t_k^-) = \lim_{m \to 0} u(t_k - m)$ represent the right and left limits of $u(t)$ at $t = t_k$, respectively.

The motivations of this paper are two aspect : on the one hand, we observed that most of the existing articles (see, [13–15,17–19]) are only devoted to study the local existence of mild solutions for fractional evolution equations, up until now the existence of mild solution for an impulsive fractional functional integro differential equation with non-compact semi-group in Banach spaces has not been considered in the literature. In order to fill this gap, we are concerned with the existence of local mild solutions and global mild solutions for the problem (1) in this paper; on the other hand, we find that the fractional evolution equations have been extensively studied in recent years by using various fixed point theorems when the corresponding semi-group $T(t)(t \geq 0)$ is compact, this is very convenient to the equations with compact resolvent. But for the case that the corresponding semi-group $T(t)(t \geq 0)$ is non-compact, there are very few papers which studied these equations, only Wang et al. [20] discussed the local existence of mild solutions for nonlocal problems of fractional evolution equations under the situation that $T(t)(t \geq 0)$ is an analytic semi-group of uniformly bounded linear operators. In this article, by using the famous Sadovskiis fixed point theorem (see Lemma 2.5) and a new estimation technique of the measure of non-compactness (see Lemma 2.6), we study local and global existence of mild solution for the problem (1). Our results can be considered as a contribution to these nascent fields.

The paper is organized as follows: In Section 2, we introduce some notations and recall some basic known results. In Section 3 we present the existence of local solutions to the problem (1) in Banach space. In Section 4 we discuss the existence theorem of global solutions for the problem (1). In Section 5, we give two examples to illustrate our results.

2. Preliminaries

Throughout this paper, we assume that A: $D(A) \subset X \to X$ is a closed linear operator and $-A$ generates a uniformly bounded C_0-semi-group $T(t)(t > 0)$ on a Banach space $(X, \|\cdot\|)$. Let I denote a closed subset of the interval $[0, +\infty)$ and let $M =$