Commun Nonlinear Sci Numer Simulat 42 (2017) 275–284

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Influence of nonlinearity on transition curves in a parametric pendulum system

Bin Zhen[a], Jian Xu[b], Zigen Song[c,*]

[a] School of Environment and Architecture, University of Shanghai for Science and Technology, Shanghai 200093, PR China
[b] School of Aerospace and Mechanics Engineering, Tongji University, Shanghai 200092, PR China
[c] College of Information Technology, Shanghai Ocean University, Shanghai 201306, PR China

ARTICLE INFO

Article history:
Received 23 January 2016
Revised 12 April 2016
Accepted 18 May 2016
Available online 30 May 2016

Keywords:
Parametric pendulum
Nonlinear Mathieu equation
The energy method
The asymptotic method

ABSTRACT

In this paper transition curves and periodic solutions of a parametric pendulum system are calculated analytically by employing the energy method. In previous studies this problem usually was dealt with by using the asymptotic method which is limited by small parameter. In our research, the hypothesis of small number in the pendulum system is not necessary, some different conclusions are obtained on the impacts of nonlinearity in the pendulum system on the transition curves in the parametric plane. The results based on the asymptotic method suggested that nonlinearity in the pendulum system only significantly causes decrease of the area of the stable regions in the parametric plane when the angular displacement of the pendulum is not very small. However, our analysis according to the energy method shows that nonlinearity does not significantly change the area of the stable regions in the parametric plane, but notably alter positions of the stable regions. Furthermore, position of the stable regions to a large extent is related to the amplitude of periodic vibrations of the pendulum especially when the angular displacement of the pendulum is large enough. Our results are very different from that reported in previous studies, which have been verified by numerical simulations.

1. Introduction

The parametric pendulum is one of the most common models with numerous engineering applications, such as marine structures, auto suspension systems, damping devices in civil engineering, energy harvesting systems, and so on. The parametric pendulum, especially with nonlinearity, has been of great interest in the past decades for its rich dynamical behavior [1], including equilibrium points, oscillations, rotations as well as chaos. Trueba et al. [6] investigated the dynamic of a pendulum under different types of harmonic excitations. Symmetry-breaking of oscillations, locating the oscillatory orbits and escape zones of the parametric pendulum have been studied by Bishop and Clifford [1,2]. Xu et al. [14] have analytically, numerically and experimentally considered the rotational motion of the harmonically excited pendulum, and obtained the regions of the rotational motion of the pendulum in the parametric plane of the frequency and amplitude of the harmonic

* Corresponding author.
E-mail address: zigensong@163.com (Z. Song).

http://dx.doi.org/10.1016/j.cnsns.2016.05.020
1007-5704/

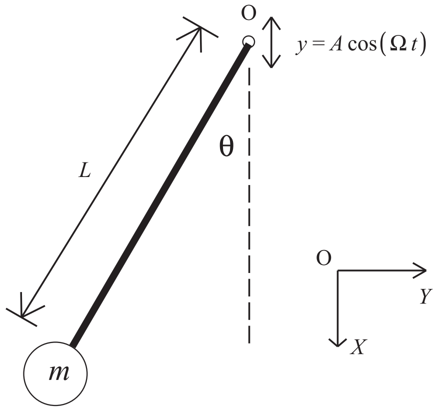

Fig. 1. The parametric pendulum diagram.

excitation. Horton et al. [7] found that an additional small horizontal motion has a significant influence on the stable periodic rotation region of the pendulum under the vertical excitation. Chaotic motion of a pendulum subjected to rotation and vertical support vibration has been analyzed by Ge et al. [3,4]. Szemplinska-Stupnicka et al. [11–13] studied the oscillation-rotations and tumbling chaos through examining their basins of attraction and regions of existence in the parameter space.

For many engineering applications, oscillatory responses of the pendulum subjected to harmonic parametric excitation **are of main interests**, which have been dealt with in number of studies. A parametric pendulum excited by vertical harmonic is shown in Fig. 1, in which m is the mass of the pendulum, L is the length of the pendulum rod. In addition, the support point O has a vertical harmonic motion, $y = A\cos(\Omega t)$, where A and Ω represent the amplitude and frequency of the support vibrating periodically respectively. The damping ratio is small enough to be neglected. Xu et al. [15] and Mann et al. [16] employed the multiple scales method to analytically investigate the periodic motion of the parametric pendulum. Sofroniou and Bishop [10] and Miles [17] studied the motion of the pendulum by using the harmonic balance method. Koch et al. [8,9] indicated the generation of periodic oscillations and rotations of the pendulum by applying Melnikov and averaging methods to identify the boundaries of subharmonic and homoclinic bifurcations.

Almost all works in the literature on the resonance boundary of the parametric pendulum were limited to small parameter. For example, the ratio between the amplitude of the support vibration and the length of the pendulum rod (A/L) was usually assumed to be far less than 1 in the study of transition curves and periodic solutions. Such condition was key to calculate the transition curves in the parametric plane and approximately periodic expressions in almost all the literature. Thus the results of the computation should be valid only for the parameters of which the ranges are far less than 1. However, before reaching resolution no one knows the ranges of interest for related parameters. One outstanding problem is that the ranges of stability charts in almost all the literature obtained by the asymptotic method which is dependent on small parameter were conflict with the hypothesis of small parameters. In other words, the ranges of the parameters showed in the stability charts did not satisfy the condition that the values of parameters were far less than 1. On the other side, nonlinearity is significant once the angular displacement of the parametric pendulum is not very small. Previous studies according to the asymptotic method considered that the positions of the transition curves of the parametric pendulum with a not very small angular displacement have nothing to do with the amplitude of the angular displacement, and nonlinearity in the pendulum system makes the stable regions decrease in the parametric plane [15]. It is worth noting that such conclusions have been drawn under the hypothesis of small parameter, which maybe not reasonable when the amplitude of the periodic vibrations of the pendulum is large enough.

This paper is concerned with the analytical investigation of periodic motion of the pendulum under vertical parametric excitation (shown as in Fig. 1). In our research, the hypothesis of small parameter in the pendulum system is not necessary. Since all asymptotic methods dependent on small parameter failure, we employ the energy method [5] to calculate the transition curves and periodic solutions of the parametric pendulum. Numerical simulations are carried out to verify the correction of our analytical results, and the influence of the amplitude of the periodic vibrations of the pendulum on the transition curves is discussed analytically and numerically. Our research shows that the value of the amplitude of the periodic vibration has a serious impact on the distribution of the transition curves of the pendulum system, especially on the position of the intersection of different transition curves in the parametric plane.

The rest of the paper is organized as follows: in Section 2 the governing equation of the parametric pendulum model is established. The transition curves and periodic solutions of the parametric pendulum are analytically obtained by using the energy method in Section 3. Numerical simulations are performed to demonstrate the correction of our analytical results in Section 4. The influence of the amplitude of the periodic vibration of the pendulum on the transition curves is discussed in Section 5. Conclusions are concluded in Section 6.