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Research paper

An explicit closed-form analytical solution for European options under the CGMY model

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ABSTRACT

In this paper, we consider the analytical pricing of European path-independent options under the CGMY model, which is a particular type of pure jump Lévy process, and agrees well with many observed properties of the real market data by allowing the diffusions and jumps to have both finite and infinite activity and variation. It is shown that, under this model, the option price is governed by a fractional partial differential equation (FPDE) with both the left-side and right-side spatial-fractional derivatives. In comparison to derivatives of integer order, fractional derivatives at a point not only involve properties of the function at that particular point, but also the information of the function in a certain subset of the entire domain of definition. This "globalness" of the fractional derivatives has added an additional degree of difficulty when either analytical methods or numerical solutions are attempted. Albeit difficult, we still have managed to derive an explicit closed-form analytical solution for European options under the CGMY model. Based on our solution, the asymptotic behaviors of the option price and the put-call parity under the CGMY model are further discussed. Practically, a reliable numerical evaluation technique for the current formula is proposed. With the numerical results, some analyses of impacts of four key parameters of the CGMY model on European option prices are also provided.

1. Introduction

It is well known that the log-increments of the underlying price are assumed to be Gaussian under the Black-Scholes (B-S) model. This assumption is, however, quite controversial, as suggested by many empirical studies [2,4,12,16]. As pointed out in [12], the distribution of underlying returns is more leptokuric than the normal distribution. This feature is more accentuated when the holding period becomes shorter and particularly clear on high frequency data. In the last decade, several alternative asset models with non-Gaussian log-increments are introduced. Those models have either added a stochastic volatility component, such as the Heston model [13], or included jumps in the underlying evolution, such as the Press model [20] and the Merton jump diffusion model [18].

A better alternative to the above two classes of models is the so-called CGMY process introduced by Carr et al. in [4]. This process is a particular type of pure jump Lévy process with four key parameters *C, G, M* and *Y* controlling its essential characteristics. By taking on proper values, the four parameters further allow diffusions and jumps to have both finite

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and infinite activity and variation [4]. Thus, the CGMY process synthesizes the features of many continuous-time models and captures their essential differences in parametric cases [4]. Empirically, this process agrees well with many observed properties of the market data [12]. As Carr et al. pointed out in [4], the statistical and risk-neutral processes for equity prices are pure jump processes with infinite activity and finite variation, which can be well characterized by the CGMY model.

In the literature, a number of authors have concentrated on the pricing of option derivatives under the CGMY model. Since the seminal work of Carr and Madan [5], many researchers applied the Fourier transform method to evaluate the prices of option derivatives. For example, Martin [17] summarized the use of Fourier transform in the valuation of options under many complicated models including the CGMY model. Based upon the FFT (fast Fourier transform) method, Almentral and Oosterlee [1] solved numerically the prices of European and American options under the CGMY model. It should be remarked that a solution written in terms of the inverse Fourier transform without the inversion being carried out analytically is of closed form. But, this kind of solution is not truly “explicit” because the numerical inversion of Fourier transform still needs to be handled carefully. Seeking an explicit closed-form analytical solution is the goal of the current work. Recently, Cartea and del-Castillo-Negrete [7] showed that option prices under the CGMY model are governed by a fractional partial differential equation (FPDE) with two spatial fractional derivatives capturing the non-locality induced by pure jumps in the underlying price. This work is the first that relates the pricing of options under the CGMY model to FPDEs. Mathematically, these FPDEs are a class of second-order Voterra partial integro-differential equations with weak singular kernels, which are difficult to solve, either analytically or numerically.

In the quantitative finance area, two types of fractional derivatives are well documented, namely, a time-fractional derivative and a spatial-fractional derivative. Typical work in the first category includes the derivations of closed-form analytical solutions for European vanilla options and double barrier options under the modified B-S equation with a time-fractional derivative [11,21], and the establishment of models containing information of waiting-time between trades using Caputo fractional derivative [6,8]. In the latter category, the most substantial progress is made by Cartea and del-Castillo-Negrete [7] by successfully connecting the finite moment log-stable (FMLS) model, the KoBol model and the CGMY model to FPDEs. They also considered the pricing of barrier options under these FPDEs by using a finite difference approach. Recently, Chen et al. [9] derived a closed-form analytical solution for European put options under the FMLS model. They also considered the pricing of American options under the FMLS model by using a predictor-corrector approach [10] introduced in [22].

In this paper, we consider the pricing of European path-independent options under the CGMY model. Our work is not a trivial extension of [9] due to the complexity of the CGMY model. In terms of the derivation of the closed-form analytical solution, the current work is totally different and much more complicated than the one did in [9], because the FPDE considered now has both the left-side and right-side fractional derivatives, with the product of exponential function and option price as the new function to be differentiated, whereas the governing FPDE of the FMLS model only has a single left-side fractional derivative. Albeit difficult, we still manage to derive an explicit closed-form analytical solution for the CGMY model. In terms of numerical implementation, the current formula is written in terms of double integrals whereas the one in [9] is in a single integral form. Furthermore, in addition to the treatment of the fox function as used in [9], a new scaling technique is introduced to effectively control the growth rate of the integrand so that a convergent numerical result can be produced.

The paper is organized as follows: In Section 2, we introduce the CGMY model and the FPDE governing the price of European options. In Section 3, we derive a closed-form analytical solution from the established FPDE system, and examine the asymptotic behaviors of the solution. We also derive the put-call parity under the CGMY model. In Section 4, numerical examples and some analyses are presented. Concluding remarks are given in the last section.

2. The CGMY model

Under a risk neutral measure $\mathbb{Q}$, the CGMY model assumes that the log of the underlying, i.e., $x_t = \ln S_t$, follows a geometric Lévy process as

$$dx_t = (r - \nu)dt + dL_t,$$

where r is the risk-free interest rate and ν is a convexity adjustment so that S_t is a martingale. For the CGMY model, $\nu = C\Gamma(-Y)\left[(M-1)^Y - M^Y + (G+1)^Y - G^Y\right]$. dL_t here is the CGMY process controlled by four parameters C, G, M and Y. As pointed out in [4], the parameter C is a measure of the overall level of activity. G and M control the rate of exponential decay on the right and left of the Lévy density, respectively. When they are equal to each other, the distribution of the CGMY model becomes symmetric. In the case of $G < M$, it leads to a skewed distribution with a heavy left tail. The parameter Y determines whether the CGMY model has a complete monotone Lévy density, and whether the process has finite or infinite activity or variation [4]. In the current work, we assume that $Y \in (1, 2)$ so that the CGMY process is completely monotone, and has an infinite variation and finite quadratic variation. The extension of our solution procedure to other values of Y are quite promising.