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Research paper

Almost structure-preserving analysis for weakly linear damping nonlinear Schrödinger equation with periodic perturbation

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ABSTRACT

Exploring the dynamic behaviors of the damping nonlinear Schrödinger equation (NLSE) with periodic perturbation is a challenge in the field of nonlinear science, because the numerical approaches available for damping-driven dynamic systems may exhibit the artificial dissipation in different degree. In this paper, based on the generalized multi-symplectic idea, the local energy/momentum loss expressions as well as the approximate symmetric form of the linearly damping NLSE with periodic perturbation are deduced firstly. And then, the local energy/momentum losses are separated from the simulation results of the NLSE with small linear damping rate less than the threshold to insure structure-preserving properties of the scheme. Finally, the breakup process of the multi-soliton state is simulated and the bifurcation of the discrete eigenvalues of the associated Zakharov–Shabat spectral problem is obtained to investigate the variation of the velocity as well as the amplitude of the solitons during the splitting process.

1. Introduction

To embody the volatility in the classical mechanics model for the quantum system, Schrödinger proposed a partial differential equation (PDE) to describe the time evolution of the quantum states for the physical system in 1926 [1], which was named as Schrödinger equation and initiated the quantum mechanics. In the past several decades, there were a large number of contributions concentrated on the solutions and the dynamic behaviours of some kinds of the Schrödinger equation [2–16].

Caldeira and Leggett moved the conservative structure of microscopic quantum mechanics into the dissipative macroscopic domain in 1983 [17]. Kostin showed the existence of the term added into the Hamiltonian of the time-dependent Schrödinger equation to remove energy without changing the wave-function normalization [18]. Unfortunately, the simulation on the dissipative quantum mechanics is difficult, in that, the rigorous treatment of dissipative term in the quantum domain needs to consider a large number of degrees of freedom, each of which is governed by a dissipative PDE [19]. Thus, to avoid the onerous numerical computation, most of the current studies on the damping nonlinear Schrödinger equation (NLSE) focused on the qualitative properties by various analytical methods [20–25].

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The representative jobs on the numerical analysis of damping NLSE include: Peranich revealed the reason for that the occurrence of the sawteeth oscillations superimposed on the smooth wave solutions of the numerical solutions presented by Delfour et al., [26] for the linear damping NLSE [27]; Zhang and Lu analysed the long-time stability and the convergence of some finite difference schemes for the weakly damping Schrödinger equation, and proved the existence of global attractor [28,29]; Bao and Jaksch introduced an extension of the time-splitting sine-spectral method for solving damping NLSEs and proved that the numerical method proposed was explicit, unconditionally stable, and time transversal invariant [30]; Asadzadeh investigated a streamline diffusion-based discontinuous Galerkin approximation for the numerical solution of the coupled damping NLSEs and extended the resulting method to a multiscale variational scheme [31].

But, avoiding or reducing the artificial dissipation in the simulation even for low-dimensional dissipative PDEs is a challenge for numerical methods. In this field, the symplectic schemes for Birkhoffian systems [32] and the conformal symplectic integrators [33] may result in some numerical schemes for the damping NLSE with tiny artificial dissipation. Focusing on the local geometric properties of the system, an almost structure-preserving strategy with tiny artificial dissipation will be proposed to investigate the dynamic behaviours of the following perturbed damping NLSE in this work,

$$i\partial_t u + \alpha\partial_{xx}u + \beta|u|^2u + i\gamma u = h(x,t) \tag{1}$$

where, u is a complex wave function, $i=\sqrt{-1}$, α, β, γ are real constants, $h(x, t)$ is the external small perturbation considered as a periodic function of time t and coordinate x.

According to the physical situations where the NLSE applied, the damping rate γ can be classified as two types [34]: One is the collisional damping that will be considered in this paper, where the damping is independent of the wave number and the damping rate γ can be assumed as a small positive number; another is the Landau damping, on a Maxwellian electron distribution, of Langmuir waves, the specific form of which was presented in Ref. [35] and limitations of which were pointed out in Ref. [34].

In the case of interactions between protein molecules and a weak laser field, the small periodic perturbation is proportional to the first-order derivative of the wave function with respect to coordinate x, hence, it can be formulated as $h(x,t)=\eta\partial_x u e^{i(k_0x-\omega_0t)}$, where η is a small real constant, k_0, ω_0 are real constants. For the propagating soliton excited by a vertical oscillation, the small periodic perturbation is directly proportional to the wave function, therefore, it can be formulated as $h(x,t)=\eta u e^{i(k_0x-\omega_0t)}$ [21]. The dynamic behaviours considered in this work are associated with the propagation of the multisoliton. At the same time, the handling processes of these two cases in the numerical method employed in this paper are similar. Thus, we will just focus on the latter case in this paper.

For the conservative NLSE without external perturbation, there were various numerical methods developed in the last century. Among which, the multi-symplectic method proposed by Bridges in 1997 [36] is remarkable, because it has paid great attention to the local geometric properties as well as the high-accuracy of the numerical schemes for the conservative NLSE in the Hamiltonian framework: Bridges presented the multi-symplectic structure and several local conservation laws of the NLSE based on the variational principle, which initiated the numerical investigation on the local geometric properties of the completely integrable NLSE [36]; furthermore, Islas and Schober developed a multi-symplectic scheme for the 1D NLSE and investigated the conservative properties of which [37]; Sun and Qin derived a six-point scheme for the coupled 1D NLSE, and proved that it is equivalent to the multi-symplectic Preissmann integrator [38]; subsequently, Aydin and Karasozen generalized the results presented in Ref. [38] to the integration of coupled NLSEs with periodic plane wave solutions [39]; Hong and his collaborators applied the Runge–Kutta–Nyström (RKN) methods to NLSEs with variable coefficients and proved that the scheme constructed by RKN method is multi-symplectic [40]; Recently, Song et al. proposed the semi-explicit multi-symplectic splitting method [41] and the multi-symplectic wavelet collocation method [42] for NLSE.

For the introduction of the damping term into the NLSE, the numerical method employed should without artificial dissipation or just with tiny artificial dissipation. In our precious jobs [43,44], we have developed the generalized multi-symplectic method with tiny artificial dissipation to record the dissipative effect in each time step and investigate the local dynamic behaviours of the weak damping dynamic systems. In this paper, the generalized multi-symplectic method will be employed to study the local dynamic behaviours of the damping NLSE with small periodic perturbation, including the losses of energy/momentum and the breakup phenomena of the multisoliton state, which will broaden the applied range of the generalized multi-symplectic method to the damping complex PDE systems.

2. Generalized multi-symplectic formulations of damping NLSE

The multi-symplectic form of the conservative NLSE has been derived from the Noether symmetry theory in 1997 [36], which can be generalized to the damping NLSE (1) naturally. In this section, the generalized multi-symplectic conservation law and the modified local conservation laws of energy/momentum will be deduced in detail. In this way, the effects of the damping and the periodic perturbation on the approximate local conservation laws of the NLSE (1) can be formulated explicitly. With these approximate local conservation laws, the structure-preserving characteristics of the numerical scheme constructed in the next section can be verified in the numerical experiments.