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Research paper

Global stability of trajectories of inertial particles within domains of instability

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ABSTRACT

Finite sized particles exhibit complex dynamics that differ from that of the underlying fluid flow. These dynamics such as chaotic motion, size dependent clustering and separation can have important consequences in many natural and engineered settings. Though fluid streamlines are global attractors for the inertial particles, regions of local instability can exist where the inertial particle can move away from the fluid streamlines. Identifying and manipulating the location of the so called stable and unstable regions in the fluid flow can find important applications in microfluidics. Research in the last two decades has identified analytical criteria that can partition the fluid domain into locally stable and unstable regions. In this paper, we identify two new mechanisms by which neutrally buoyant inertial particles could exhibit globally stable dynamics in the regions of the fluid flow that are thought to be locally unstable and demonstrate this with examples. The examples we use are restricted to the simpler case of time independent fluid flows.

1. Introduction

It is well known that particles with a finite inertia moving in a fluid can exhibit complex dynamics such as chaotic motion and size dependent clustering and separation. Understanding and having the ability to predict such phenomena in many natural settings, for instance the motion of plankton in the oceans and that of spores and pollutants in the atmosphere [1,2], can have important consequences. In the engineered setting, the ability to manipulate the dynamics of inertial particles can be very useful in emerging microfluidic applications such as the identification and separation of particles or cells by inertial properties [3–7], through purely hydrodynamic means. The nontrivial dynamics of particles in these examples arise due to the finite inertia of the particle, which allows a particle to have a non zero velocity relative to the fluid and thus a trajectory different from the underlying fluid flow, [8–11].

Early work on inertial particle dynamics was on the settling of particles under gravity in a quiescent fluid [12–14]. This was later extended to particles settling under gravity in unsteady but uniform flows and finally to unsteady and non-uniform flows [8,15]. The Maxey–Riley equations offer a minimal model to incorporate the effects of finite size of a spherical rigid particle in an unsteady and non uniform flow. Michaelides [16] gives a good historical review of the development of this equation. Several investigations on the dynamics of active and passive inertial particles were performed by Tel, Toroczai, Benczik and others where they investigated the finite size effects of particles in various autonomous and non-autonomous flows [17, [18–21]]. From these investigations, it becomes apparent that there exist some regions of a flow field where

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the finite size effects of inertial particles cause them to have dynamics that are significantly different from that of passive tracers. Babiano et.al [22] and later, Haller and Sapsis [23] have obtained analytical criteria to identify regions of the flow where the inertial particle trajectories deviate significantly from the passive tracer trajectories. A review of these and other recent advances in the nonlinear dynamics of finite sized particle in a fluid are described in [24].

These analytical criteria [22,23] identify regions of a fluid flow field, where particles of a certain size are likely to lose their velocity relative to the underlying flow and start moving with the fluid streamlines. It is worth emphasizing at this point that such regions which act as attractors for particles of one size may not remain an attractor for a particle of a different size. These criteria are particularly useful in studies of microfluidic flows for engineering applications where size based differential behavior of particles can be exploited for a variety of applications ranging from identification of particle type in a mixture to effective particle mixing or the opposite case of size based particle separation. Being able to predict regions of any flow field where particles of a certain size are going to cluster and hence, focus into thin bands, enables us to design microfluidic channels which take advantage of this behavior to separate out particles based on size [7]. On the contrary, identifying flow configurations that do not have such clear attracting regions for particles helps us design better particle mixers in cases where a homogeneous mixture of different sized particles is the desired outcome.

The criteria derived in [22,23] partition the fluid domain into unstable regions where the velocity of inertial particles relative to the fluid increases and thus the particles deviate from the fluid flow and stable regions where the velocity of inertial particles relative to the fluid decays. The criteria in [22,23] determine the local stability of subsets of the fluid domain with the results in [22] being restricted to a time independent flow. In the case of time dependent fluid flows the results in [23] establish the existence of a slow manifold to which inertial particles converge to. These results have however often been interpreted as results on global stability to identify regions where inertial particles cluster (or are attracted to). In this paper we use two simple examples of time independent fluid flows which dramatically illustrate that the locally unstable sets in the fluid domain could in fact be global attractors for inertial particles. We show that neutrally buoyant particles can converge towards fluid streamlines even in regions of the fluid domain which are identified as unstable. We identify two possible mechanisms that lead to such behavior.

In the broader context of nonlinear dynamics our findings in this paper describe a novel type of an invariant set that contains locally unstable subsets which repel nearby trajectories, but with trajectories converging back to the unstable set. However the local unstable sets have an interesting feature. The global unstable manifold that emanates from each point of the unstable subset intersects non transversally with the local stable manifold of another point the in unstable set. Thus the locally unstable invariant sets become globally stable. To our knowledge, only one other source mentions the existence of a 'flow' which is locally unstable everywhere but is globally stable, [25]. However this example occurs in a spatio temporal system based on the Ginzburg–Landau equation.

This paper is organized as follows: In Section 2, we review the equation governing inertial particle dynamics, namely the Maxey–Riley equation [8] and its reduced form which we will use in our numerical calculations. In Section 3, we review some existing analytic criteria in literature which helps us identify regions of a flow domain which are expected to act as repellers to the trajectories of particles with finite relative velocities in phase space. In Section 4, we present counter examples to demonstrate the existence of a mechanism by which the relative velocity of an inertial particle may decay in unstable regions of the flow as identified by the criteria in [22,23].

2. Governing equations

The equation of motion of a small, spherical, rigid particle in an incompressible fluid, following the work of Maxey and Riley [8] is

$$\rho_p \frac{d\mathbf{v}}{dt} = \rho_f \frac{D\mathbf{u}}{Dt} + (\rho_p - \rho_f)g - \frac{9\nu\rho_f}{2a^2}\left(\mathbf{v} - \mathbf{u} - \frac{a^2\nabla^2\mathbf{u}}{6}\right) - \frac{\rho_f}{2}\left(\frac{d\mathbf{v}}{dt} - \frac{D}{Dt}\left[\mathbf{u} + \frac{\mathbf{a}^2}{10}\nabla^2\mathbf{u}\right]\right)$$
$$- \frac{9\rho_f}{2a}\sqrt{\frac{\nu}{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\frac{d}{d\tau}\left(\mathbf{v} - \mathbf{u} - \frac{a^2\nabla^2\mathbf{u}}{6}\right)\partial\tau \tag{1}$$

Here ρ_p and ρ_f are the densities of the particle and fluid respectively, $\mathbf{v}$ is the velocity of the inertial particle, $\mathbf{u}$ is the velocity of the fluid at the location of the particle, g is the acceleration due to gravity, ν is the kinematic viscosity and a is the particle radius. Denoting the length scale and the velocity scale of the fluid flow by L and U respectively, the regime of validity of (1) is where the particle size is small compared to the length scale of the fluid flow, $\frac{a}{L} \ll 1$ and the particle Reynolds number is small, $(\frac{a^2}{\nu})(\frac{U}{L}) \ll 1$. The first term on the right hand side of (1) is the force exerted on the particle by the undisturbed flow. The second term is the buoyancy force and the third is the Stokes drag. The fourth term is the added mass correction and the final term is the Basset–Boussinesq history force.

Some common simplifications can be made to (1) by considering a restricted range of parameters and flow conditions. The particle radius a is chosen such that a^2 is sufficiently small compared to the characteristic length scale of the flow (L) so that the Faxen correction term $a^2\nabla^2\mathbf{u}$ can be neglected. The Basset history term which includes viscous memory effects into the equation is negligible when the relative acceleration of the particle is small and it is negligible when the fluid flow is steady and at low Reynolds number, [16]. Recent research [26,27] suggests that the memory term is negligible in comparison to inertial terms only if both the Stokes number and the fluid Reynolds numbers are small. Our analysis and examples are