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Research paper

Torsional instability in suspension bridges: The Tacoma Narrows Bridge case

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All attempts of aeroelastic explanations for the torsional instability of suspension bridges have been somehow criticised and none of them is unanimously accepted by the scientific community. We suggest a new nonlinear model for a suspension bridge and we perform numerical experiments with the parameters corresponding to the collapsed Tacoma Narrows Bridge. We show that the thresholds of instability are in line with those observed the day of the collapse. Our analysis enables us to give a new explanation for the torsional instability, only based on the nonlinear behavior of the structure.

1. Introduction

Some suspension bridges manifested aerodynamic instability and uncontrolled oscillations leading to collapses, see e.g. [1,29]. These accidents are due to several different causes and the focus of this paper is to analyse those due to wide torsional oscillations. Thanks to the videos available on the web [50], many people have seen the spectacular collapse of the Tacoma Narrows Bridge (TNB) occurred in 1940. The torsional oscillations were considered the main cause of the collapse [2,48]. But the appearance of torsional oscillations is not an isolated event occurred only at the TNB: among others, we mention the collapse of the Brighton Chain Pier in 1836, the collapse of the Wheeling Suspension Bridge in 1854, the collapse of the Matukituki Suspension Footbridge in 1977. We refer to [25] for a detailed description of these collapses.

These accidents raised some fundamental questions of deep interest for both engineers and mathematicians. Due to the vortex shedding, longitudinal oscillations are to be expected in suspension bridges, but the reason of the sudden transition from longitudinal to torsional oscillations is less clear. So far, no unanimously accepted response to this question has been found. Most attempts of explanations are based on aeroelastic effects such as the frequency of the vortex shedding, parametric resonance, and flutter theory. The purpose of the present paper is to show that the origin of the torsional instability is purely structural.

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Von Kármán, a member of the Board appointed for the Report [2], was convinced that the torsional motion seen on the day of the collapse was due to the vortex shedding that amplified the already present torsional oscillations and caused the center span to violent twist until the collapse, see [18, p.31]. But Scanlan [47, p.841] proved that the frequency of the torsional mode had nothing to do with the natural frequency of the shed vortices following the von Kármán vortex pattern: the calculated frequency of a vortex caused by a 68 km/h wind is 1 Hz, whereas the frequency of the torsional oscillations measured by Farquharson (an engineer witness of the TNB collapse, the man escaping in the video [50]) was 0.2 Hz, see [13, p.120]. The conclusion in [13, p.122] is that the vortex trail is a consequence, not a primary cause of the torsional oscillation. Also Green–Unruh [27, Section III] believe that vortices form independently of the motion and are not responsible for the catastrophic oscillations of the TNB. The vortex theory was later rediscussed by Larsen [34, p.247], who stated that vortices may only cause limited torsional oscillations, but cannot be held responsible for divergent large-amplitude torsional oscillations. Recently, McKenna [40] noticed that the behavior described in Larsen's paper was never observed at the Tacoma Bridge and also Green–Unruh [27] believe that *the Larsen model does not adequately explain data or simulations at around 23 m/s*.

Bleich [14] suggested a possible connection between instability in suspension bridges and flutter of aircraft wings; but Billah–Scanlan [13, p.122] believe that it is a great mistake to relate these two phenomena. Billah–Scanlan also claim that their own work proves that the failure of the TNB was in fact related to an aerodynamically induced condition of self-excitation in a torsional degree of freedom; but Larsen [34, p.244] believes that the work in [13] fails to connect the vortex pattern to the switch of damping from positive to negative. Moreover, McKenna [40] states that [13] *is a perfectly good explanation of something that was never observed, namely small torsional oscillations, and no explanation of what really occurred, namely large vertical oscillations followed by torsional oscillations*.

The parametric resonance method was adapted to a TNB model by Pittel–Yakubovich [42,43], see also [51, Chapter VI] for the English translation and a more general setting. The conclusion on [51, p.457] claims that *the most dangerous phenomenon for the stability of suspension bridges is a combination of parametric resonance*. But Scanlan [47, p.841] comments these attempts by writing that *Others have added to the confusion. A recent mathematics text [51], for example, seeking an application for a developed theory of parametric resonance, attempts to explain the Tacoma Narrows failure through this phenomenon*. We refer to [15,28,31] for connections between (aerodynamic) parametric resonance, flutter theory, and self-oscillations, also applied to suspension bridges.

To conclude this quick survey of attempts for aeroelastic explanations, we mention that Scanlan [46, p.209] writes that *...the original Tacoma Narrows Bridge withstood random buffeting for some hours with relatively little harm until some fortuitous condition "broke" the bridge action over into its low antisymmetrical torsion flutter mode*; the words *fortuitous condition* tell us that no satisfactory aeroelastic explanation is available. Due to all these controversial discussions, McKenna [39, Section 2.3] writes that *there is no consensus on what caused the sudden change to torsional motion*, whereas Scott [48] writes that *opinion on the exact cause of the Tacoma Narrows Bridge collapse is even today not unanimously shared*. Summarizing, all the attempts to find a purely aeroelastic explanation of the TNB collapse fail either because the quantitative parameters do not fit the theoretical explanations or because the experiments in wind tunnels do not confirm the underlying theory.

Nowadays the attention has turned to the nonlinear behavior of structures [3,33]. In a recent paper [4] we gave an explanation in terms of a *structural instability*: we considered an isolated nonlinear bridge model and we were able to show that, if the longitudinal oscillations are sufficiently small, then they are stable whereas if they are larger they can instantaneously switch to destructive torsional oscillations. The main tool used there are transfer maps (Poincaré maps), which highlight an instability when the characteristic multipliers exit the complex unit circle. The same phenomenon was later emphasized also for different models using results on the instability for the Hill equation and Floquet theory, see [8–10]. All these results were obtained by considering isolated systems, that is, by neglecting both the aerodynamic forces and the dissipation. The idea to consider an isolated system was already suggested by Irvine [30, p.176] for a suspension bridge model similar to the one considered in [4]: Irvine ignores damping of both structural and aerodynamic origin, his purpose being to simplify as much as possible the model by maintaining its essence, that is, the conceptual design of bridges. And since our purpose is precisely to highlight the role of the structure on the stability, we follow this suggestion: in the conclusions of the present paper we mention how the aerodynamic effects combine with the structural effects.

In this paper we improve the results in [4] by introducing a more refined model which also takes into account the nonlinear restoring action of the cables+hangers system on the deck; the model is described in detail in Section 2 and is the nonlinear version of a linear model that we introduced in [5]. In Section 3 we define the nonlinear longitudinal modes, that is, the periodic purely longitudinal motions of the deck which may be unstable and create torsional motions. In Sections 4.1 and 4.2 we describe the theoretical framework that governs the stability of the longitudinal modes and we prove that they are stable for small energies. Finally, we validate our theory by performing numerical simulations of the model with the parameters of the collapsed TNB. We study the existence and the behavior of approximate longitudinal modes and we compute their instability thresholds. Our numerical results confirm that the longitudinal oscillations with eight or nine nodes and amplitudes of about 4 m, namely the oscillations observed the day of the collapse, are prone to generate torsional oscillations, see Section 4.3. The main conclusions and our explanation of the TNB collapse are collected in Section 5.