Commun Nonlinear Sci Numer Simulat 42 (2017) 502–519

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Dynamics of the higher-order rogue waves for a generalized mixed nonlinear Schrödinger model

Lei Wang[a,*], Dong-Yang Jiang[b], Feng-Hua Qi[c], Yu-Ying Shi[a], Yin-Chuan Zhao[a]

[a] Department of Mathematics and Physics, North China Electric Power University, Beijing 102206, P. R. China
[b] School of Nuclear Science and Engineering, North China Electric Power University, Beijing 102206, P. R. China
[c] School of Information, Beijing Wuzi University, Beijing 101149, P. R. China

ARTICLE INFO

Article history:
Received 15 January 2016
Revised 17 April 2016
Accepted 9 June 2016
Available online 14 June 2016

PACS:
05.45.Yv
42.50.Md
42.65.Tg
42.81.Dp

Keywords:
Generalized mixed nonlinear schrödinger equation
Modulational instability
Darboux transformation
Rogue waves
Semirational solutions

ABSTRACT

Under investigation in this paper is a generalized mixed nonlinear Schrödinger equation (GMNLSE) which arises in several physical areas including the quantum field theory, weakly nonlinear dispersive water waves, and nonlinear optics. The linear stability analysis is performed and the instability zones as well as the modulational instability gain are obtained and discussed. Higher–order rogue waves (RWs) in terms of the determinants for the GMNLSE model are constructed by the *N*-fold Darboux transformation. Several patterns of the RWs are illustrated, such as the fundamental pattern, triangular pattern, circular pattern, pentagon pattern, circular–triangular pattern, and circular-fundamental pattern. Effects of the nonlinear parameters on the RWs are discussed. It is found that the nonlinear terms affect the widths and velocities of the RWs, although the amplitudes of these waves remain unchanged. The semirational RW solution, which is a combination of rational and exponential functions, is derived to describe the interaction between the RW and multi-breather.

1. Introduction

In recent years, the studies of rogue waves (RWs) have attracted considerable attention because of their potential applications in various fields including the oceanography [1], nonlinear fiber optics [2], Bose-Einstein condensates [3], atmospheric dynamics [4], plasma [5], laser-plasma interactions [6], and even finance [7]. In contrast to ordinary solitons which are the stable waves, RWs are the localized structures with the instability and unpredictability [8,9]. The generation mechanism of the RWs is Benjamin–Fier instability [10], also referred to as modulational instability (MI). Mathematically, they can be expressed as the rational functions which are viewed as the limit of either Kuznetsov-Ma breathers (KMBs) [11] or Akhmediev breathers (ABs) [12], or as homoclinic orbits of unstable Stokes waves under periodic boundary conditions [13]. Optical RWs with higher amplitudes indicate their potential applications in transmitting highly intense signals through optical fibers and open the doors for their use in digital communication. Some progress on the optical RWs have been obtained, such as their utility in telecommunication data streams [14], the manipulation of optical RWs in nonlinear fibers [15], the generation of optical RWs in higher–order equations [16], and RWs in optically injected lasers [17], to name a few.

* Corresponding author.
E-mail address: wanglei2239@126.com, 50901924@ncepu.Edu.cn (L. Wang).

http://dx.doi.org/10.1016/j.cnsns.2016.06.011
1007-5704/

Currently, there are several techniques for investigating exact solutions such as the Darboux transformation (DT) [18], bilinear method [19] and some direct search approaches [20]. Based on the Lax pair, the DT is used to obtain the determinant forms of soliton solutions of some nonlinear physical models [18,21]. Advantage of the DT is that the problem of solving a nonlinear physical model is finally reduced to solve two linear systems, i.e., two linear partial differential equations [22] given by Lax pair. However, in order to obtain the RW solutions, some techniques in DT need to be modified. Instead of the trivial seeds, the plane–wave solutions are used for producing the breather solutions [22,23]. Further, by means of the Taylor expansion technique, one can derive the RW solutions [22,23]. In addition, the bilinear method, another effective method in deriving the multi–soliton solutions, has likewise been applied to the rational (or RW) solutions of many nonlinear physical models [24–26], especially the high–dimensional ones [25].

Among various models for describing the RWs in nonlinear optical fibers, the nonlinear Schrödinger (NLS) equation

$$i\,q_t + q_{xx} + 2\,|q|^2\,q_x = 0\,, \tag{1}$$

is the most accepted one, which admits the Peregine breather as a popular candidate for the RWs [27]. Nevertheless, in optic fiber communication systems, one always has to increase the intensity of the incident light field to produce ultrashort (femtosecond) optical pulses [28]. In this case, the standard NLS equation is inadequate to accurately model the phenomena, which leads to such more subtle effects as the higher–order nonlinear terms, third-order dispersion, self–steepening, and self–frequency shift adopted [29–32]. Additionally, adding the variable coefficients can result in novel propagation behaviors of RWs [15,31–33].

In this paper, we will consider the following generalized mixed nonlinear Schrödinger equation (GMNLSE) [34–36] in the form of

$$i\,\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \pm i\,a\,|u|^2\,\frac{\partial u}{\partial x} \pm i\,b\,u^2\,\frac{\partial u^*}{\partial x} + c\,|u|^4\,u \pm d\,|u|^2\,u\,, \tag{2}$$

where the asterisk means the complex conjugate, $u = u(x,t)$ is the complex function of $(x,\ t)$, and a, b, c and d are all real constants. Eq. (2) arises in several physical applications including the quantum field theory, weakly nonlinear dispersive water waves, and nonlinear optics [34–36]. It is shown to enjoy the Painlevé property only if $c = \frac{1}{4}\,b\,(2\,b - a)$ holds, regardless of the value of d [34–36]. With different choices of the multiple parameters a, b, c and d, a series of celebrated nonlinear evolution equations in mathematical physics are included by Eq. (2) [35–49]. Recently, the N-fold DT and soliton solutions with multi–parameters have been constructed with the help of gauge transformation [50]. The bright and dark (including gray– and black–soliton) envelope solutions have also been derived within the framework of the Madelung fluid description [51]. However, to the best of our knowledge, very little attention has been paid to the study of the higher-order RWs for Eq. (2). Note that the RWs of the Kundu-Eckhaus (KE) equation that also includes quintic nonlinear term have been obtained in [52]. But Eq. (2) cannot be reduced to the KE equation, because the value of a is not equal to that of b in the Lax pair (see Eq. (3) in [50]). It is worthy of paying attention to Eq. (2) since it covers abundant nonlinear models of physical and/or mathematical interest.

The outline of this paper will be organized as follows: The MI will be discussed in Section 2; N-fold DT of a generalized Ablowitz–Kaup–Newell–Segur (AKNS) spectral problem will be constructed in Section 3; Higher–order RW solutions depending on a certain number of parameters for Eq. (2) will be obtained in Section 4; Structures of the higher–order RWs and effects of the nonlinear terms will be discussed through figures in Section 5; The semirational RW solution will be derived in Section 6; Our conclusions will be given in Section 7.

2. Modulational instability

Based on the method of investigating the MI of the NLS and coupled NLS equations [53], we hereby perform a linear stability analysis of the plane-wave solution for Eq. (2) [the case " − "] under the constraint

$$c = \frac{b(2b - a)}{4}\,. \tag{3}$$

It is easy to find that Eq. (2) admits the exact continuous-wave solution in the form of plane wave,

$$u(t,x) = q\,\exp[i(\omega t + sx)]\,, \tag{4}$$

where ω is a real parameter, and

$$\omega = \frac{1}{4}\left(abq^4 + 4aq^2 s - 2b^2 q^4 - 4bq^2 s - 4dq^2 + 4s^2\right). \tag{5}$$

Substitution of the perturbation solution

$$\psi(t,x) = (q + \epsilon\,\Phi(t,x))\,\exp[i(\omega t + sx)]\,, \tag{6}$$