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Research paper

Stability of solitary waves for the generalized nonautonomous dual-power nonlinear Schrödinger equations with time-dependent coefficients

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ABSTRACT

In this paper, we explicitly obtain the solitary waves of the generalized nonautonomous dual-power nonlinear Schrödinger equations (DPNLS) with time-dependent coefficients. The stability of the solitary waves of the nonautonomous DPNLS is strictly proved by Vakhitov-Kolokolov stability criterion. Finally, we show solitary waves of three interesting examples to test our stability conditions.

1. Introduction

The nonlinear Schrödinger equations (NLS) is one of the most important nonlinear models of modern science, whose extensions have extensively attracted much attention due to their numerous physical applications in many branches of nonlinear science, such as nonlinear optics [1,5,17,27,30,36,42,51] and the Bose-Einstein condensates (BECs)[14,24,34]. Recently, a nonautonomous nonlinear Schrödinger system with variable coefficients has attracted a lot of attention because of its interesting features and potential applications [23,29,38,45]. Further, many researchers studied a generalized nonautonomous NLS equation with the dual-power(DP) nonlinearities in nonlinear optics, when the intensity of the optical pulse propagating inside nonlinear medium exceeds a certain value or the two-and-three-body interactions in BECs are considered, see Biswas and co-workers [1–8,15–17,26,32,39–43,50–52], the others see [25,46,49] and the references therein.

In this paper, we will consider solitary waves of the 1D generalized nonautonomous dual-power nonlinear Schrödinger equations (DPNLS) with time-dependent coefficients

$$iQ_t + D(t)Q_{xx} + (lR_1(t)|Q|^n + kR_2(t)|Q|^{2n})Q + V(x,t)Q = 0, \tag{1}$$

where $Q(x, t)$ is the complex envelope of the propagating beam of the modes, x represents the propagation distance, and t represents the retarded time. n, l, k are arbitrary real constants, the functions $D(t)$ represent dispersion, $lR_1(t)|Q|^n + kR_2(t)|Q|^{2n}$ represent the dual-power nonlinearity and $V(x, t)$ represents the external potential. In [38] shows that the basic property of classical solitary waves, to interact elastically, holds true for Eq. (1). However, some novel feature arises for Eq. (1), both amplitudes and speeds of the solitary waves, consequently their spectra during the propagation and after the

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interaction are no longer the same as those prior to the interaction. Thus, it is valuable and necessary to study the stability of solitary waves of (1) under different time-dependent coefficients.

In [20–22,26], similarity transformation was applied to obtain solitary waves for some kinds of nonlinear Schrödinger equations or systems. Firstly in this paper, using similarity transformation, we deduce the generalized nonautonomous DPNLS to the following usual autonomous DPNLS equation

$$iq_T + q_{XX} + (l|q|^n + k|q|^{2n})q = 0. \tag{2}$$

Generally speaking, Eq. (2) is non-integrable, expect we set $l = 1, n = 2, k = 0$, when it is classical (NLS). Otherwise, (2) was also called a dual-power medium equation, which describes the saturation of the nonlinear refractive index, and also serves as a basic model to describe the solitons in photovoltaic-photorefractive materials such as LiNbO3 [2,26,39,43], nano-photonics and nano-optics [6–8,15,17,32,39,42,43,50–52]. Concretely, let $n = 2, k = 0, l$ is any constants, dual-power law collapses to the Kerr law nonlinearity [2,3]; let $l = 0$, n, k are any constants, dual-power law collapses to the power law nonlinearity [8,31]; let $n = 2$, l, k are any constants, dual-power law collapses to the parabolic law nonlinearity [40,50,52], which is also called cubic-quintic law nonlinearity. Secondly, through some calculation, we get stationary solitary waves of the usual autonomous DPNLS (2), then using Galilean invariance, we can obtain moving solitary waves of (2). Finally using similarity transformation, we could obtain solitary waves of Eq. (1). These obtained solutions may be useful to exhibit exactly the nonlinear wave propagations through optical fibers with non-Kerr law nonlinearity[4,16,25,43,48,49] and to test the accuracy of numerical solutions. These analyses followed up with stability studies.

Linear stability of solitary waves of the nonlinear Schrödinger equations (NLS) is an important issue in nonlinear science, which has numerous physical applications. Vakhitov-Kolokolov(VK) stability criterion is widely used in the analysis of stability for the solitary waves for the (NLS) and other similar systems [10,13,18,44]. Later, some generalized VK stability criterion was obtained in [46]. Recently, Other stability criterion for solitary waves of the generalized nonlinear Schrödinger equation (NLS) has been conjectured in [35]. However, as far as we know, stability of solitary waves for the generalized nonautonomous DPNLS (1) hasn't been proved strictly. In Section 3 of our paper, we perturb the solitary waves of the generalized nonautonomous DPNLS, and obtain stability criterion for solitary waves of the DPNLS (1) by (VK) criterion as [44,46]. The solitary waves of the generalized nonautonomous DPNLS (1) is linearly unstable if and only if $P'(\mu) < 0$, where $\mu > 0$ and

$$P(\mu) = \int |Q(x, t, \mu)|^2 dx, \tag{3}$$

where Q is solitary waves of (1), and P is called power or charge, depending on the model under consideration. These results are all supported with very meaningful and superb numerical simulation. In Section 4, choosing different coefficients $D(t)$, $R_1(t)$, $R_2(t)$, $V(x, t)$ and different parameters k, l, n, we show three interesting examples with physical applications.

2. Solitary waves of nonautonomous DPNLS equation

2.1. The similarity transformation

In this section, we choose proper similarity transformations in order to map generalized nonautonomous DPNLS (1) into the usual autonomous DPNLS Eq. (2). The similarity transformation is introduced as

$$Q(x, t) = q(X, T)p(t)e^{i\phi(x,t)}, \tag{4}$$

where X, T, p, ϕ can be expressed by x and t. We can find the advantages of transformation (4) is to solve generalized nonautonomous DPNLS by using solutions of the usual autonomous DPNLS equation, which is much easier to treat.

The aim of this section is to find the specific expressions about D, R_1, R_2, $V(x, t)$ by designing real smooth functions X, T, p, ϕ and at the same time $q(X, T)$ need satisfy the autonomous DPNLS (2). Take (4) into (1), we obtain

$$\begin{aligned} &ie^{i\phi(x,t)}(pq_X X_t + pq_T T_t + p_t q + ipq\phi_t) + De^{i\phi(x,t)}(pq_{XX}X_x^2 + pq_X X_{xx} + 2ipq_X X_x \phi_x - pq\phi_x^2 \\ &\qquad + ipq\phi_{xx}) + (lR_1|pqe^{i\phi(x,t)}|^n + kR_2|pqe^{i\phi(x,t)}|^{2n})pqe^{i\phi(x,t)} + Vqpe^{i\phi(x,t)} = 0, \end{aligned} \tag{5}$$

In order to make (5) coincident with (2), let

$$D = \frac{T_t}{{X_x}^2}, \quad R_1 = \frac{T_t}{p^n}, \quad R_2 = \frac{T_t}{p^{2n}}. \tag{6}$$

Take (6) into (5), we have

$$\begin{aligned} &iq_T + q_{XX} + (l|q|^n + k|q|^{2n})q + \left(\frac{iq_X X_t}{T_t} + \frac{iqp_t}{pT_t} - \frac{q\phi_t}{T_t}\right) \\ &\quad + \left(\frac{2iq_X\phi_x}{X_x} + \frac{q_X X_{xx}}{X_x^2} - \frac{q\phi_x^2}{X_x^2} + \frac{iq\phi_{xx}}{X_x^2}\right) + \frac{V(x, t)q}{T_t} = 0, \end{aligned} \tag{7}$$