Commun Nonlinear Sci Numer Simulat 42 (2017) 571–584

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Vegetation pattern formation of a water-biomass model☆

Xiaoli Wang, Wendi Wang*, Guohong Zhang

Key Laboratory of Eco-environments in Three Gorges Reservoir Region, School of Mathematics and Statistics, Southwest University, Chongqing, 400715, PR China

ARTICLE INFO

Article history:
Received 23 October 2015
Revised 17 January 2016
Accepted 6 June 2016
Available online 7 June 2016

Keywords:
Reaction-diffusion system
Cross-diffusion
Turing pattern
Pattern solution

ABSTRACT

In this paper, a mathematical model with diffusion and cross-diffusion is proposed to describe the interaction between the vegetation and the soil water. Based on the view of Turing pattern, we discuss the conditions of the diffusion-induced instability and the cross-diffusion-induced instability of a homogenous uniform steady state. We find that either a fast diffusion speed of water or a great hydraulic diffusivity due to the suction of roots may drive the instability of the homogenous steady state. Furthermore, we find that both the rain-fall rate and the infiltration feedback parameter can induce the transitions among the vegetation state, pattern formation and bare soil state. It is also found that the "terrain slope" may cause the instability of the homogenous steady state and drive the formation of periodic stripe pattern. Consequently, the diversity of dryland vegetation in reality can be explained as a result of pattern solutions of the model.

1. Introduction

In the past four decades, the ecological unbalance between the limited water resource and ecosystem engineers [14,15], such as animals, plants or microorganisms, has been observed and the desertification has become more and more serious [10]. The desertification may be a slow and gradual process in which the vegetation is not homogenous but self-organized spatial patterns [3,25,30]. The vegetation spatial patterns exhibit repetitive or distinctive patches in space and time. Their forms can be a two-phased irregular mosaic consisting of a high-cover phase and a low-cover phase (bare land) [2] or many different regular types such as bands, labyrinth, spots, stripes, gaps, and rings [16,17,23,24,30,31,35], which are observed in arid and semiarid areas.

How ecosystem engineers affect ecosystems and how the vegetation consists of patterns are the main frontiers in ecology [30] and have fascinated many ecologists [14,15,40]. Several mechanisms have been elucidated [40] to underly patterning of vegetation. Basically, these mechanisms include water scarcity, plant competition over water resources, redistribution of water by diffusion and runoff, and the positive feedback between water availability and plant [24,28,31,33,39]. Other factors, such as livestock overgrazing [29], climate variables [1,6,8], soil properties [38], rainfall interception [13,32], and toxic compounds [4,21,22] may also drive the formation of patterns. Recently, mathematical modeling has been shown to be a powerful approach to understand mechanisms of pattern formations [10,16,18,30,31,35,36,39]. Based on the view of vegetation pattern formation as a symmetry-breaking phenomenon [5], the Klausmeier's model [16] generates the complex spatial patterns, often called a Turing pattern [37], from a relatively simple dynamical system.

☆ This work is supported by grants from National Science Foundation of China (11571284, 11461023).

* Corresponding author.
E-mail address: wendi@swu.edu.cn (W. Wang).

http://dx.doi.org/10.1016/j.cnsns.2016.06.008
1007-5704/

To understand the mechanism for generation of vegetation patterns and their observed resilience, Shnerb et al. [35] propose the following mathematical model of one species (shrubs or trees) and one resource (water):

$$\begin{cases} \dfrac{\partial w}{\partial t} = D_w \Delta w + \mathbf{v} \cdot \triangledown w + R - \lambda wb - w, \\ \dfrac{\partial b}{\partial t} = wb - \mu b, \\ w(x,0) = w_0(x), b(x,0) = b_0(x), \end{cases} \tag{1.1}$$

where w is the ground water density and b is the shrubs biomass density; parameters D_w, R, λ, μ are positive constants that denote the water diffusion coefficient, the rain-fall rate, the water consumption rate in the presence of vegetation and the vegetation death rate, respectively; the term $\mathbf{v} \cdot \triangledown w$ takes account of the downhill water loss in which $\mathbf{v}$ is a "terrain slope" vector. The initial states $w_0(x)$ and $b_0(x)$ are nonnegative functions of spatial variable x. The initial value problem of (1.1) is subject to a spatially periodic boundary condition over a planar spatial domain.

In [35], Shnerb et al. have shown that the final state of (1.1) is a uniform covering of all the plane by the amount of flora that corresponds to the stable fixed point. Note that there is no cross-diffusion in (1.1). Note also that the water infiltration at vegetation is important [9,10]. Following [11,19,39], we modify the self-diffusion of water in (1.1) to $D_w\Delta(w - \beta b)$, where $\beta > 0$ represents the hydraulic diffusivity due to the suction of roots in the vadose zone. To capture the "infiltration feedback" between the plant and the ground water [9,10], following [20] we assume that the vegetation death rate μ is monotonously decreasing in b and is described by

$$\mu(b) = \mu_0 + \frac{\mu_1}{b+1}, \tag{1.2}$$

where μ_0, μ_1 are positive constants. Biologically, it can be explained as follows. The plant can loosen the soil locally which in turn increases the infiltration of the vegetation patch. Thus, the larger plant biomass density results in the higher infiltration, which decreases the death rate of shrubs due to the more soil water available. As a consequence, we consider the following model:

$$\begin{cases} \dfrac{\partial w}{\partial t} = D_w \Delta (w - \beta b) + v\dfrac{\partial w}{\partial x_1} + R - \lambda wb - w, & x \in \Omega,\ t > 0, \\ \dfrac{\partial b}{\partial t} = D_b \Delta b + wb - \mu(b) b, & x \in \Omega,\ t > 0, \\ \dfrac{\partial w(x,t)}{\partial \nu} = \dfrac{\partial b(x,t)}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ w(x,0) = w_0(x) \ge 0, b(x,0) = b_0(x) \ge 0, & x \in \Omega. \end{cases} \tag{1.3}$$

where all quantities are in nondimensional form; Ω is a bounded planar domain with a smooth boundary $\partial\Omega$; no-flux boundary condition is imposed on $\partial\Omega$ so that the ecosystem is closed to exterior environment; ν is the outward unit normal vector of the boundary $\partial\Omega$; $x = (x_1, x_2)$ is the spatial variable; $\Delta = \frac{\partial}{\partial x_1^2} + \frac{\partial}{\partial x_2^2}$ is the Laplacian operator; the diffusion term $D_b\Delta b$ denotes the spread of plants both by clonal reproduction and by seed dispersal [26], where D_b is a positive constant. Moreover, the surface runoff is modeled by the term $v\frac{\partial w}{\partial x_1}$, where v is a constant downhill runoff flow velocity in the negative x_1–direction [16].

This paper is organized as follows. In Section 2, we analyze the diffusion-induced instability. In Section 3, we consider the cross-diffusion-induced instability. The effect of the ground surface is shown in Section 4. We end with discussions in Section 5.

2. Diffusion-induced instability

We start to consider the diffusion-induced instability, which means that the uniform steady state loses its stability due to the diffusion effect. When $\beta = 0$ and $v = 0$, we have the following initial boundary problem

$$\begin{cases} \dfrac{\partial w}{\partial t} = D_w \Delta w + R - \lambda wb - w, & x \in \Omega,\ t > 0, \\ \dfrac{\partial b}{\partial t} = D_b \Delta b + wb - \left(\mu_0 + \dfrac{\mu_1}{b+1}\right) b, & x \in \Omega,\ t > 0, \\ \dfrac{\partial w(x,t)}{\partial \nu} = \dfrac{\partial b(x,t)}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ w(x,0) = w_0(x) \ge 0, b(x,0) = b_0(x) \ge 0, & x \in \Omega. \end{cases} \tag{2.1}$$

Here, we consider system (2.1) in the spatial domain $\Omega = (0, l\pi)$. Firstly, we present the following lemma about the global stability of the bare-soil steady state $(w_0, b_0) = (R, 0)$, which means the extinction of the plant.

Lemma 2.1. *Let the parameters D_w, D_b, R, λ, μ_0, μ_1 be positive. Then the bare-soil steady state (w_0, b_0) of (2.1) is globally asymptotically stable if $R \le \mu_0$.*