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Existence and exponential stability for neutral stochastic integrodifferential equations with impulses driven by a fractional Brownian motion☆

G. Arthi [a,b], Ju H. Park [a,*], H.Y. Jung [a]

[a] Department of Electrical Engineering/Information and Communication Engineering, Yeungnam University, 280 Daehak-Ro, Kyongsan 38541, Republic of Korea
[b] Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore, Tamil Nadu 641004, India

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ABSTRACT

In this paper, we establish the results on existence and uniqueness of mild solution of impulsive neutral stochastic integrodifferential equations driven by a fractional Brownian motion. Further, by using an impulsive integral inequality, some novel sufficient conditions are derived to ensure the exponential stability of mild solution in the mean square moment. The results are obtained by utilizing the fractional power of operators and the semigroup theory. Finally, an example is presented to demonstrate the effectiveness of the proposed result.

1. Introduction

In many real-world applications, one adopts that the system under deliberation is governed by a principle of causality. That is, the future state of the system is independent of the past states and is determined exclusively by the present. However, under closer analysis, it turn into evident that the principle of causality is often only a first approximation to the exact situation and that a more realistic model would contain some of the past states of the system. Stochastic functional differential equations are frequently used to describe a mathematical formulation for such systems. A class of dynamical systems depends on past as well as present values but which include derivatives with delays as well as the function itself. Such systems have been referred as neutral functional differential equations. The study of deterministic neutral functional differential equations was initiated by Hale and Meyer [17]. For more details on theory and their applications, we also refer the readers to Hale and Verduyn Lunel [18], Kolmanovskii and Nosov [20] and so on. In recent years, the quantitative and qualitative properties of solutions to stochastic and

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* Corresponding author.Tel.: +82-53-8102491.
E-mail addresses: arthimath@gmail.com (G. Arthi), jessie@ynu.ac.kr (J.H. Park).

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stochastic neutral differential equations like existence, uniqueness and stability have been widely examined by many researchers due to various mathematical models in the different areas such as mechanics, electronics, control theory, engineering, economics, etc. (see [1,5,7,10,16,23,24,26,27,29,36,37,40,41] and the references therein).

Fractional Brownian motion is a Gaussian stochastic process, which depends on a parameter $H \in (0, 1)$ called the Hurst index and it was introduced by Kolmogorov [21]. For additional details on the fractional Brownian motion, we refer the reader to [28]. This stochastic process has self-similarity, stationary increments and long-range dependence properties. In the particular case $H = 1/2$, the fractional Brownian motion reduces to standard Brownian motion. But for $H \neq 1/2$, fractional Brownian motion is not a semimartingale. Recently, the study of differential equations driven by a fractional Brownian motion has been investigated by many authors, see, for example, [2,3,14,30,31,34,43] and the references therein.

On the other hand, impulsive differential equation is an emerging field drawing attention from both theoretical and applied disciplines. It provides a natural description of systems and generally describes the phenomena which is subject to abrupt or instantaneous changes. Indeed, we can find numerous applications in physics, mechanical, electrical, economics and several fields in engineering , see [22,38] and the references therein. It is well-known that the notion of " aftereffect" introduced in physics is very important. To model processes with aftereffect, it is not sufficient to employ ordinary or partial differential equations. An approach to resolve this problem is to use integrodifferential equations. Integrodifferential equations arise in many engineering and scientific disciplines, often as approximations to partial differential equations which represent much of the continuum phenomena. Recently, the theory and application of impulsive stochastic integrodifferential equations have received much attention. But only a few papers deal with the stability of mild solutions to stochastic integrodifferential equations with impulsive effects [6,13,19,25,39].

In this paper, we consider the following neutral stochastic integrodifferential equation with impulses of the form:

$$d\left[x(t) - g\left(t, x_t, \int_0^t a_1(t, s, x_s)ds\right)\right] = \left[Ax(t) + f\left(t, x_t, \int_0^t a_2(t, s, x_s)ds\right)\right]dt + \tilde{F}(t)dB_Q^H(t), \ t \in [0, b], \ t \neq t_i, \tag{1}$$

$$\triangle x(t_i) = I_i(x(t_i^-)), \ t = t_i, \ i = 1, 2, \ldots, \tag{2}$$

$$x_0(t) = \phi(t) \in \mathcal{PC}([-r, 0], X), \ -r \leq t \leq 0, \tag{3}$$

where A is the infinitesimal generator of an analytic semigroup $(T(t))_{t \geq 0}$ of bounded linear operators in a Hilbert space X, B_Q^H is the fractional Brownian motion, $g, f : [0, +\infty) \times \mathcal{PC} \times X \to X, a_1, a_2 : [0, +\infty) \times [0, +\infty) \times \mathcal{PC} \to X, \tilde{F} : [0, +\infty) \to L_Q^0(Y, X)$ are appropriate functions and will be specified later. The impulsive moments t_i satisfy $0 < t_1 < t_2 < \ldots < t_i < \ldots$, and $\lim_{i \to +\infty} t_i = \infty$. I_i: $X \to X$, $\triangle x(t_i)$ represents the jump in the state x at t_i determining the size of the jump, which is defined by $\triangle x(t_i) = x(t_i^+) - x(t_i^-)$, where $x(t_i^+)$ and $x(t_i^-)$ are respectively the right and the left limits of $x(t)$ at t_i. $\mathcal{PC} = \{\phi : [-r, 0] \to X, \phi(t)$ is continuous everywhere except a finite number of points $\tilde{t}$ at which $\phi(\tilde{t}^-)$, $\phi(\tilde{t}^+)$ exist and $\phi(\tilde{t}^-) = \phi(\tilde{t})\}$. For $\phi \in \mathcal{PC}$, $\|\phi\|_{\mathcal{PC}} = \sup_{s \in [-r, 0]} \|\phi(s)\| < +\infty$. For any continuous function x and any $t \in [0, b]$, we denote by x_t the element of $\mathcal{PC}$ defined by $x_t(\theta) = x(t + \theta)$, $-r \leq \theta \leq 0$.

In [42], Taniguchi et al. investigated the existence, uniqueness and asymptotic behavior of mild solutions to stochastic partial functional differential equations. The exponential stability in the pth moment of mild solutions to impulsive stochastic neutral partial differential equations with memory has been discussed by Yang and Jiang [44]. In [8], Chen studied the asymptotic behavior for second-order neutral stochastic partial differential equations with infinite delay. Ferrante and Rovira [15] examined the existence and uniqueness of solution for stochastic differential delay equations with fractional Brownian motion for $H > 1/2$. Caraballo et al. [4] established the existence, uniqueness and exponential asymptotic behavior of mild solutions to stochastic delay evolution equations perturbed by a fractional Brownian motion. Boufoussi and Hajji [3] proved an existence, uniqueness and exponential decay to zero in mean square moment for the mild solutions to a neutral stochastic differential equation driven by a fractional Brownian motion in a Hilbert space.

Very recently, in [32], Nguyen has obtained a sufficient condition for the exponential asymptotic behavior of solutions of a class of linear fractional stochastic differential equations with time-varying delays and Chen et al. [9] studied the exponential stability of impulsive neutral stochastic partial functional differential equations. Dung [12] investigated the mild solutions of neutral stochastic differential equations driven by a fractional Brownian motion with impulsive effects and delays. The stochastic volterra integro-differential equations driven by fractional Brownian motion in a Hilbert space have been discussed in [13]. However, up to now, existence and stability problems for neutral stochastic integrodifferential systems with impulses driven by a fractional Brownian motion have not been considered in the literature. Therefore, it is necessary to consider the impulsive effects to stability of solutions of neutral stochastic integrodifferential equations driven by a fractional Brownian motion. To the best of our knowledge, there is no work which examines the study of existence and exponential stability for impulsive neutral stochastic integrodifferential equations driven by a fractional Brownian motion and the intention of this paper is to close this gap.

The rest of this paper is arranged in the following way. In Section 2, some necessary notations and concepts are provided. In Section 3, the results on existence and uniqueness of mild solutions are established. Section 4 is devoted to the proof of exponential stability of a mild solution in mean square moment, followed by an illustrative example in Section 5. At last, the conclusion is given in Section 6.