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Research paper

Implicit Euler approximation of stochastic evolution equations with fractional Brownian motion

Minoo Kamrani[a,b,*], Nahid Jamshidi[a]

[a] Department of Mathematics, Faculty of Sciences, Razi University, 67149 Kermanshah, Iran
[b] School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P. O. Box:19395-5746, Tehran, Iran

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ABSTRACT

This work was intended as an attempt to motivate the approximation of quasi linear evolution equations driven by infinite-dimensional fractional Brownian motion with Hurst parameter $H > \frac{1}{2}$. The spatial approximation method is based on Galerkin and the temporal approximation is based on implicit Euler scheme. An error bound and the convergence of the numerical method are given. The numerical results show usefulness and accuracy of the method.

1. Introduction

Models based on stochastic partial differential equations (SPDEs) play a prominent role in a range of application areas, including chemistry, physics, biology, economics, finance, microelectronics, and nowadays also nanotechnology.

Very recently, stochastic equations in infinite dimensions with fractional Brownian motion as a driving process has become an important subject of interest of many researchers. These objects are interesting because they model a type of random disturbances with features different from those of Wiener process (like the long-range dependence). They also provide new mathematical challenges because large parts of the standard stochastic analysis tools are not applicable for them.

Fractional Brownian motion indexed by a Hurst parameter $H \in (0, 1)$ is a Gaussian process. It has been introduced by Hurst [8] to model the long term storage capacity of reservoirs along the Nile river. This process seems to be applicable in modeling real phenomena based especially in empirical data such as economic data (see e.g. [3,9]) telecommunication traffic (see e.g. [13,18]) and others.

Since a fractional Brownian motion with $H \neq \frac{1}{2}$ is not a semi martingale, the standard stochastic calculus and the classical stochastic integration theory cannot be used for them. In recent years there have been various developments of stochastic calculus for this process, especially for $H \in (\frac{1}{2}, 1)$(see e.g. [1,2,19]). One of the main obstacles in the stochastic calculus for fractional Brownian motion is the concept of stochastic integral which has been discussed by [1,19] and [6].

Finite dimensional stochastic differential equations driven by fractional Brownian motion have been treated for instance in [14] and [16]. In [10] for these kinds of equations, existence and uniqueness results are given. Theory of SPDEs driven by fractional Brownian motion also has been studied recently. For example Linear and semilinear stochastic equations in a Hilbert space with an infinite dimensional fractional Brownian motion are considered in [4] and [5].

* Corresponding author.
E-mail address: m.kamrani@razi.ac.ir, minookamrani@gmail.com (M. Kamrani).

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Numerical solutions of stochastic differential equations (SDEs) driven by fractional Brownian motion have been investigated via several schemes such as Milestein and Euler scheme by [11] and [15] . But numerical approximation for SPDEs with fractional Brownian motion have been very little studied. In [12] the explicit Euler scheme for SPDEs with fractional Brownian motion has been proposed and the rate of convergence of the approximation method is established.

But as we know there are lots of equations which explicit methods are not applicable for them, for example for stiff systems in many cases we have to concentrate on implicit methods. The main idea of this article is to propose an implicit method for solving numerical solution of SPDEs driven by infinite dimensional fractional Brownian motions. For this aim for spatial discretization we apply Galerkin method and for time discretization the implicit Euler scheme will be used and then the convergence rate of the method will be proved.

The rest of this article is organized as follows. In Section 2 we define the fractional Brownian motion and the stochastic fractional integral. The basic setting and the assumptions are presented in Section 3 and in Subsection 3.1 a brief exposition of Galerkin Method is introduced. The numerical scheme and its rate of convergence theorem which are the main results of this article are given in Section 4 . Finally in Section 5 a method(Cholesky methods) for simulating of fractional Brownian motions is described and the main result of the paper is illustrated with some examples.

2. Preliminary

In this section, we review some of the standard facts on the fractional calculus. Let $T \in (0, \infty)$ be a real number and $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space. Assume $U = (U, \langle .,.\rangle, |.|_U)$ and $V = (V, \langle .,.\rangle, |.|_V)$ are separable Hilbert spaces and $\mathcal{L}_2 := \mathcal{L}_2(U, V)$ be the family of linear Hilbert-Schmidt operators from U to V with $|T|_{\mathcal{L}_2} = (\sum_{k=1}^{\infty} |Te_k|^2)^{\frac{1}{2}}$, where $\{e_k, k \in \mathbb{N}\}$ is a set of complete orthonormal basis in U. The following definition provides an infinite-dimensional analogue of a fractional Brownian motion in a finite-dimensional space with Hurst parameter $H \in (\frac{1}{2}, 1)$[4].

Definition 2.1. A $U-$valued Gaussian process $(B^H(t), t \in \mathbb{R})$ on $(\Omega, \mathcal{F}, \mathbb{P})$ is called a fractional Brownian motion with Hurst parameter $H \in (\frac{1}{2}, 1)$ if

(i) $E(B_t^H) = 0$,
(ii) $E(B_t^H B_s^H) = \frac{1}{2}\{|t|^{2H} + |s|^{2H} - |t-s|^{2H}\}$.

If $H = \frac{1}{2}$, then the corresponding fractional Brownian motion is the usual standard Brownian motion.

A standard (cylindrical) fractional Brownian motion with the Hurst parameter $H \in (\frac{1}{2}, 1)$ in the Hilbert space U is defined by

$$B^H(t) = \sum_{n=1}^{\infty} \beta_n^H(t) e_n, \tag{2.1}$$

where $\{\beta_n^H(t), n \in \mathbb{N}, t \in \mathbb{R}\}$ is a sequence of independent, real-valued standard fractional Brownian motions each with the same Hurst parameter H.

Let $p > \frac{1}{H}$ be an arbitrary but fixed number and G be a function such that for each $x \in U$, $G(.)x \in L^p([0, T], V)$ and

$$\int_0^T \int_0^T |G(s)|_{\mathcal{L}_2} |G(r)|_{\mathcal{L}_2} |s-r|^{2H-2} dsdr < \infty, \tag{2.2}$$

then the stochastic integral is defined as

$$I(G, 0, T) = \int_0^T G dB^H = \sum_{n=1}^{\infty} \int_0^T G e_n d\beta_n^H. \tag{2.3}$$

The sequence of random variables $(\int_0^T G e_n d\beta_n^H, n \in \mathbb{N})$ are mutually independent Gaussian random variables, such that

$$\begin{aligned} \mathbb{E}|I(G, 0, T)|_V^2 &= \sum_{n=1}^{\infty} \mathbb{E}\Big|\int_0^T G e_n d\beta_n^H\Big|_V^2 \\ &\leq H(2H-1) \sum_{n=1}^{\infty} \int_0^T \int_0^T \langle G(s)e_n, G(r)e_n\rangle_V |s-r|^{2H-2} drds \\ &\leq H(2H-1) \int_0^T \int_0^T |G(s)|_{\mathcal{L}_2(U,V)} |G(r)|_{\mathcal{L}_2(U,V)} |s-r|^{2H-2} dsdr < \infty. \end{aligned} \tag{2.4}$$

3. Setting of the problem

Let $U = V = L^2(0, 1)$, equipped with the norm $\|.\|$ and inner product $< .,. >$. Fix γ, θ, ρ and θ_σ, such that $\gamma > 0, 0 \leq \theta \leq \frac{1}{2}, 0 \leq \rho < \min(\frac{1}{2}, \gamma)$, and $\rho + \theta_\sigma \leq \theta$.

Through the paper let the following assumptions are satisfied.