## Research paper

# Numerical studies of the KP line-solitons 

S. Chakravarty*, T. McDowell, M. Osborne<br>Department of Mathematics, University of Colorado, Colorado Springs, CO 80918, USA

## A R T I C L E I N F O

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#### Abstract

The Kadomtsev-Petviashvili (KP) equation admits a class of solitary wave solutions localized along distinct rays in the xy-plane, called the line-solitons, which describe the interaction of shallow water waves on a flat surface. These wave interactions have been observed on long, flat beaches, as well as have been recreated in laboratory experiments. In this paper, the line-solitons are investigated via direct numerical simulations of the KP equation, and the interactions of the evolved solitary wave patterns are studied. The objective is to obtain greater insight into solitary wave interactions in shallow water and to determine the extent the KP equation is a good model in describing these nonlinear interactions.


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## 1. Introduction

An important example of physically interesting nonlinear wave equations was proposed in 1970 by Kadomtsev and Petviashvili [2] in their study of plasma waves. It is a $(2+1)$-dimensional, weakly nonlinear dispersive wave equation of the form

$$
\begin{equation*}
\left(4 u_{t}+6 u u_{x}+u_{x x x}\right)_{x}+3 u_{y y}=0 \tag{1}
\end{equation*}
$$

where $x, y, t$ are the spatial coordinates and time; $u=u(x, y, t)$ represents the (normalized) wave amplitude; the subscripts denote partial derivatives. Eq. (1) is referred to as the KP equation throughout the text. We point out that (1) is the KP equation with negative dispersion, while the positive dispersion KP equation (not considered here) corresponds to (1) with a negative sign in front of the $u_{y y}$-term. Eq. (1) is derived from the three-dimensional Euler equations for an irrotational and incompressible fluid under the assumptions that it describes the propagation of small amplitude, weakly dispersive, uni-directional waves with small, transverse variation. From a physical perspective, the KP equation has been studied in the context of oblique interactions of ion-acoustic and shallow water solitary waves. An example of such wave phenomena observed in nature is the surface wave patterns created by the oblique interaction of incoming waves in shallow water on long, flat beaches as shown in Fig. 1.

Eq. (1) admits an important class of solitary wave solutions that are regular, non-decaying, and localized along distinct lines in the $x y$-plane. These solutions are known as the line-soliton solutions, which in recent years, have been a subject of extensive research that led to a complete classification of these solutions using geometric and combinatorial techniques [4,5,7]. These solutions feature an arbitrary number of asymptotic line solitons in the far-field and a complex interaction pattern of intermediate solitons resembling a web-like structure in the near-field region. Several examples of such exact solutions have been constructed in recent laboratory wave tank experiments demonstrating surface wave patterns generated by solitary wave interactions [10,13].

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Fig. 1. Beach wave patterns. Photographs by M. J. Ablowitz and D. E. Baldwin [3].


Fig. 2. One-soliton solution of the KP equation.

An important problem is to study the initial value problem of the KP equation in order to investigate two-dimensional wave patterns generated by the interaction of initial solitary waves. However, at present, there is no available analytic method to study the KP initial value problem with non-decaying initial data in the form of solitary waves in the far-field region. In this paper, we investigate such initial value problems associated with the KP equation numerically in order to gain insight into the interaction properties of the non-decaying solitary waves in shallow water. We perform direct numerical simulations of the KP equation with a variety of initial conditions, study the convergence of initial data to the exact solutions, and analyze the relation between the parameters defining the initial conditions and those of the exact solutions. We consider types of initial waves relevant to both physical problems and experiments. Finally, we compare the numerical simulations with the theoretical results derived for the KP line-solitons.

The paper is organized as follows: in Section 2, we provide a brief summary of the theoretical development for the KP line-solitons and their classification obtained earlier in [4,5]. Section 3 describes the numerical scheme implemented in this paper and the proposed method to measure how close the numerical wave patterns are to the exact KP line-soliton solutions in a local $L^{2}$ sense. In Section 4, we describe our numerical results by considering some simple forms of piecewise defined initial conditions. We take initial data consisting of two or more semi-infinite line solitons joined together, representing interacting solitary waves. Finally, in Section 5, we summarize our work and provide a brief outlook for future work.

## 2. The KP line-solitons

The simplest example of a KP line-soliton is the one-soliton solution, which is a traveling wave

$$
\begin{equation*}
u(x, y, t)=\frac{1}{2}\left(k_{2}-k_{1}\right)^{2} \operatorname{sech}^{2} \frac{1}{2}\left(k_{2}-k_{1}\right)\left[x+\left(k_{1}+k_{2}\right) y-c_{12} t-x_{12}\right] \tag{2}
\end{equation*}
$$

and is localized along a line $L_{12}: x+\left(k_{1}+k_{2}\right) y-c_{12} t-x_{12}=0$ in the $x y$-plane for fixed $t$, as shown in Fig. 2. The onesoliton is characterized by two real, distinct parameters $k_{1}<k_{2}$ which determine the soliton amplitude: $\frac{1}{2}\left(k_{2}-k_{1}\right)^{2}$, soliton speed: $c_{12}=k_{1}^{2}+k_{1} k_{2}+k_{2}^{2}$, and the soliton slope: $k_{1}+k_{2}=\tan \Psi_{12}$, where $\Psi_{12}$ is the angle, measured counterclockwise between the line $L_{12}$ and the positive $y$-axis. We usually denote a one-soliton solution as the [1,2]-soliton, since it is determined by the parameters $k_{1}$ and $k_{2}$.

The solution $u(x, y, t)$ of (1) is usually prescribed in terms of the $\tau$-function $\tau(x, y, t)$ as

$$
\begin{equation*}
u(x, y, t)=2(\ln \tau)_{x x} \tag{3}
\end{equation*}
$$

For the general line-soliton solutions of KP, the $\tau$-function is a linear combination of exponential functions whose exponents are linear in $x, y$, and $t$. Furthermore, $\tau(x, y, t)$ depends on
(i) $M$ distinct real parameters $\left\{k_{1}, k_{2}, \ldots, k_{M}\right\}$ ordered as $k_{1}<k_{2}<\cdots<k_{M}$, and

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[^0]:    * Corresponding author.

    E-mail address: chuck@math.uccs.edu (S. Chakravarty).

