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Research paper

Dual effects of buoyancy and enstrophy transfer on scaling behavior of a shell model proposed for homogeneous turbulent convection

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A B S T R A C T

In this paper, we explain why the Bolgiano-Obukhov (BO) scaling behavior is unavailable by the SabraT model proposed for turbulent thermal convection in the range of $1 < \delta < 2$, which is extended from the Sabra model by coupling temperature with velocity in the equations of motion as an external forcing, i.e., buoyancy. Numerical studies show that SabraT model is mainly governed by the enstrophy budget equation, at which the buoyancy is not always relevant to the statistical properties and the effect of buoyancy is dependent on the parameter γ that measures the ratio of enstrophy to energy. When buoyancy is important, BO scaling is expected using theoretical arguments, such as dimensional analysis. Instead of BO scaling, a new γ-dependent scaling behavior is setup in the buoyancy relevant regime, which is found to equivalently deviate from the enstrophy cascade scaling and BO scaling. This deviation is mainly discussed by two dimensionless parameters, which respectively measure the deviation of the energy/enstrophy transfer flux rate and the injected energy/enstrophy due to buoyancy from dimensional analysis. The introduced buoyancy plays as a relative small perturbed forcing on the Sabra model without changing much its intrinsical statistical properties, i.e., dimensional analysis is not always validated in both Sabra and SabraT models.

1. Introduction

Appearing in many natural phenomena and technological applications, turbulent thermal convection is a problem of great research interest [1,2], which is driven by buoyancy forces generated by temperature differences. One interesting issue is to understand the scaling behavior of the velocity and temperature fluctuations in the inertial range of scales in which buoyancy is relevant or irrelevant [3]. It has been proposed [4] that when buoyancy is dominant and temperature acts as an active scalar, the scaling behavior of velocity and temperature are governed by a direct entropy cascade of constant entropy flux with inverse energy transfer flux and would be described by the Bolgiano and Obukhov (BO) scaling [5] plus corrections [6,7] in the range containing scales between the Bolgiano length scale L_B [8], and the large scale of the system L_0, that determined by geometry. Recently, Zhou numerically verified these phenomenon by investigation of the two-dimensional (2D) Rayleigh-Taylor turbulence, where the mean kinetic energy is dynamically transferred to large scales by an inverse

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cascade, while both the mean thermal energy or entropy and the mean enstrophy move towards small scales by forward cascades [9]. When buoyancy is irrelevant and temperature acts as a passive scalar in the range of $L < L_B$, the scaling behaviors are governed by entropy and energy cascades and would be described by Kolmogorov 1941 (K41) scaling [10] plus intermittency corrections, which has been verified in the time scale by Ching [11] and Zhou [12].

In order to understand the buoyancy-driven turbulent phenomenology, many different techniques and approximations have been developed, such as shell models. Shell models focussing on the energy cascade process have been studied intensively and proved to be useful for understanding the scaling behavior of velocity fluctuations in inertia-driven turbulence (see, for example, [13] for a review). Two classes of shell models have been proposed for studying homogeneous and isotropic turbulence. The first class consists of the shell model proposed by Desnyansky and Novikov [14], in which different scaling behaviors are observed due to different attractive fixed-point solutions [15]. The other class consists of the Gledzer-Ohkitani-Yamada (GOY) model [16] and its improved model, i.e., the Sabra model [17], which is proposed to eliminate some undesirable periodic oscillation in the GOY model. In the GOY or Sabra model, the scaling behavior is dependent on model parameter δ, i.e., K41 scaling is set up in the range of $0 < \delta < 1$ for 3D turbulent flow and energy/enstrophy equipartition and cascades scalings are set up in the range of $1 < \delta < 2$ for 2D turbulent flow [18,19]. The inviscid invariants, like the energy and enstrophy, are important quantities determining the dynamics of the Sabra model.

Two classes of shell models are correspondingly constructed for turbulent thermal convection. For the first class, both BO scaling and K41 scaling behaviors have been reported in the shell model first constructed by Brandenburg [20] and also in the modified model by Suzuki and Toh [21] for some parameter range. Fruitful following theoretical and numerical results have been reported by us from the Brandenburg model, such as scalings and heat flux transfer issues for turbulent homogeneous convection [7,22,23]. For the second class, K41 scaling has been reported by Jiang and Liu [24] using a shell model extended from the GOY model. In the range of $0 < \delta < 1$, K41 scaling plus intermittency corrections is also set up by the SabraT model [25], extended from the Sabra model. Thus, buoyancy is found to be not always significant and relevant to the statistical properties in these shell models even though there is an explicit coupling term with temperature in the equation of motion for velocity [22]. Recently, the Sabra model is also modified to study other turbulent issues, apart from turbulent homogeneous convection. Based on the Sabra model, a 2D-3D shell model is constructed for investigating the phenomenology of turbulent cascades in fluid layers with large aspect ratio, namely Rayleigh-Taylor convection [26]. Besides the convective turbulence, shell models can be used to study another buoyancy-driven turbulence, such as stably stratified turbulence [27–32]. A unified shell model for stably stratified and convective turbulence is also constructed from the Sabra model to yield BO scaling for stably stratified flows and K41 scaling for convective turbulence [33]. Moreover, K41 scaling has been reported in most of these models while BO scaling is only reported in the first class of shell model, i.e., Brandenburg model with suitable parameters. In particular, there is the natural question of why BO scaling is unavailable in the second class of shell model proposed for turbulent homogeneous convection, i.e., the SabraT model.

In this paper, we focus on studying the SabraT model in the range of $1 < \delta < 2$. In Section 2, we describe the SabraT model with the Sabra model. In Section 3, we study the observed scaling behavior of velocity in the SabraT model. In this section, we show that four regimes exist with different buoyant effects. A modified enstrophy cascade scaling behavior, instead of BO scaling and enstrophy cascade scaling, is set up in regime (III). In Section 4, buoyancy effects of the SabraT model is discussed. Finally, we end this paper with a discussion and conclusion in Section 5.

2. The SabraT model

In the Boussinesq approximation, the dynamical equations that describe the homogeneous turbulent convection that driven by a constant temperature gradient along the vertical direction are

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\nabla p + \nu \nabla^2 \vec{V} + \alpha g \theta \hat{z}, \tag{1}$$

$$\frac{\partial \theta}{\partial t} + \vec{V} \cdot \nabla \theta = \kappa \nabla^2 \theta + \beta V_z \tag{2}$$

with $\nabla \cdot \vec{V} = 0$. Here, $\vec{V}$ is the velocity field, $\theta = (T - T_0 - \beta z)$ is the deviation of temperature from a linear gradient $-\beta$, T_0 is the mean temperature of the fluid, p is the pressure divided by the density, and $\hat{z}$ is a unit vector in the vertical direction. Furthermore, g is the acceleration due to gravity and α, ν, κ are, respectively, the volume expansion coefficient, the kinematic viscosity and thermal diffusivity of the fluid.

The SabraT model has been proposed for studying the homogeneous turbulent convection driven by a temperature gradient. The basic idea of shell model is to construct a set of ordinary differential equations of fluid mechanics in wave-vector representation, "shells", taking into account only a few variables per shell, such as u_n and θ_n. These variables can be roughly thought of as the Fourier transforms of the velocity and temperature fields with wavevector $\vec{k}$, whose magnitude satisfies $k_n \le |\vec{k}| \le k_{n+1}$. Here, $k_n = 2^n k_0$ is the wavenumber of the nth shell, with $0 \le n < N$, and $k_0 = 1$ is the wavenumber corresponding to the largest scale in the system.