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Research paper

Rogue waves of a $(3+1)$-dimensional nonlinear evolution equation

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ABSTRACT

General high-order rogue waves of a $(3+1)$-dimensional Nonlinear Evolution Equation ((3+1)-d NEE) are obtained by the Hirota bilinear method, which are given in terms of determinants, whose matrix elements possess plain algebraic expressions. It is shown that the simplest (fundamental) rogue waves are line rogue waves which arise from the constant background with a line profile and then disappear into the constant background again. Two subclass of nonfundamental rogue waves are analyzed in details. By proper means of the regulations of free parameters, the dynamics of multi-rogue waves and high-order rogue waves have been illustrated in (x,t) plane and (y,z) plane by three dimensional figures.

1. Introduction

The phenomena of rogue waves [1], mostly known as large and spontaneous ocean surface waves, should be responsible for a large number of maritime disasters. In the past decades, the experimental observation and theoretical analysis on rogue waves have ranged from Bose-Einstein condensates [2,3] to optical system [4–6], ocean [7], superfluids [8], plasma [9,10] and so on [11]. Mathematically, the fundamental rogue wave solution (i.e., first-order rogue wave solution) was firstly obtained in the nonlinear schrödinger (NLS) equation by Peregrine [12], which is located in both space and time, and the rogue wave solution also can be called Peregrine solution. Recently, the higher-order rogue wave solutions in NLS equation were studied in many articles [13–19]. It is seen that the higher-order rogue waves can be treat as superpositions of several fundamental rogue waves, and the superpositions can create higher amplitudes which still keep located both of space and time. What's more, the hierarchy of rogue wave solutions for other soliton equations have also been reported in references [20–29].

In addition to the one dimensional rogue wave studied so far, rogue waves in two dimensional have also derived a lot of attention. Furthermore, the two-dimensional analogue of rogue wave, expressed by more complicated rational form, has been recently reported in the Davey - Stewartson (DS) systems [30,31], Kadomtsev-Petviashvili-I equation [32,33], Yajima-Oikawa system [34], and the Fokas system [28] and so on [35,36]. As the $(3+1)$-dimensional systems also play an important role in physical systems, a natural motivation is to investigate rogue waves in $(3+1)$-dimensional model equations. Recently, Zha have derived the fundamental rogue wave in a $(3+1)$-dimensional Nonlinear Evolution Equation by a simple symbolic computation approach [37].

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In this Letter, we consider a $(3+1)$-dimensional Nonlinear Evolution Equation ($(3+1)$-d NEE)

$$3w_{xz} - (2w_t + w_{xxx} - 2ww_x)_y + 2(w_x\,\partial_x^{-1} w_y)_x = 0\,, \tag{1}$$

is a new integrable equation, was first introduced in the study of the algebraic geometrical solutions [38]. Although the application of this $(3+1)$-dimensional mode for physics or other science is not clear, but it is not hard to find that the relationship of the (3+1)-dimensional NEE (1) and the famous Kortewege de Vries (KdV) equation is very strong. The KdV equation is given by

$$u_{t'} - 6u\,u_{x'} + u_{x'x'x'} = 0\,. \tag{2}$$

Under the construction $u(x',t') \to w(x,t)\,, x' \to \frac{1}{\sqrt{3}}x\,, t' \to \frac{1}{6\sqrt{3}}t$, the KdV equation can be transformed into the main term $2w_t + w_{xxx} - 2ww_x$ possessed by the (3+1)-d NEE (1). Recently, various nonlinear types of KdV equations such as Kadomtsev - Petviashvili equation have been developed in a large range of physical phenomena. As an extension of the KdV equation, the (3+1)-d NEE (1) may be applied to model shallow water waves and short waves in nonlinear dispersive models. Except for the fundamental rogue waves obtained by Zha [37], the other solutions including solitons [39–42], positions [42] have been derived by Hirota bilinear method, Darboux transformation or some other method. So investigating the other kinds of rogue waves and the general high-order rouge waves of the $(3+1)$-d NEE is an important motivation.

In our present work, we obtain the general high-order rogue waves of the $(3+1)$-d NEE (1), which are expressed in term of determinants based on the Hirota's bilinear method [43] and KP hierarchy reduction method [44]. The basic idea is to treat $(3+1)$-d NEE as a constrained KP hierarchy. Then, we derive the rational solutions of the $(3+1)$-d NEE from the rational solutions of KP hierarchy. What's more, the rational solutions can be in a simple representation. Furthermore,we could investigate the dynamic behaviors of high dimensional rogue waves of the $(3+1)$-d NEE.

The outline of the paper is organized as follows. In Section 2, the bilinear forms and rational solutions of the $(3+1)$-d NEE will be given. In Section 3, the form of rogue waves for the $(3+1)$-d NEE is studied in details, and typical dynamics of the obtained rogue waves is analyzed and illustrated. The Section 4 contains a summary and discussion.

2. The rational solutions of (3+1)-d NEE equation

In this section, we focus on the rational solutions of the $(3+1)$-d NEE (1). Firstly, through the dependent variable transformation

$$w = -(2\log f)_{xx}, \tag{3}$$

the (3+1)-d NEE Eq. (1) can be transformed into the bilinear forms

$$(3D_xD_z - 2D_yD_t - D_yD_x^3) f\cdot f = 0. \tag{4}$$

Here f is a real function with respect to variables x, y, z and t, and the operator D is the Hirota's bilinear differential operator [43] defined by $P(D_x, D_y, D_t,)F(x,y,t\cdots)\cdot G(x,y,t,\cdots) = P(\partial_x - \partial_{x'}, \partial_y - \partial_{y'}, \partial_t - \partial_{t'}, \cdots)F(x,y,t,\cdots)G(x',y',t',\cdots)|_{x'=x,y'=y,t'=t}$, where P is a polynomial of D_x, D_y, D_t, $\cdots$.

Theorem 1. *The (3+1)-d NEE Eq. (1) has rational solutions*

$$u = -2(\log f)_{xx}, \tag{5}$$

where

$$f = \det_{1\le i,j\le N}(m_{i,j}), \tag{6}$$

and the matrix elements in f are defined by

$$m_{i,j} = \left(\sum_{k=0}^{n_i} c_{ik}(p_i\partial_{p_i} + \xi_i')^{n_i-k} \times \sum_{l=0}^{n_j} c_{jl}^*(p_j^*\partial_{p_j^*} + \xi_j'^*)^{n_j-l}\right)\frac{1}{p_i+p_j^*}, \tag{7}$$

with

$$\xi_i' = p_i x + 2\,i\,p_i^2\,y + 3\,p_i^3\,t + 4\,i\,p_i^4\,z. \tag{8}$$

Here asterisk denotes complex conjugation, and i, j, n_i, n_j are arbitrary positive integers, p_i and p_j are arbitrary complex constants.

By a scaling of $m_{ij}^{(n)}$, we can normalize $a_0 = 1$ without loss of generality, and thus hereafter set $a_0 = 1$. These rational solutions can also be expressed in terms of Schur polynomials as shown in [19,30]. What's more, the non-singularity of these rational solutions have been proof in [19,35] if the real parts of wave numbers p_i $(1 \le i \le N)$ are all positive or negative. Next, we will provide the proof of this theorem.